EGR 537–001 **NUMERICAL ANALYSIS** **Fall 2005**

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Numerical analysis is the branch of applied mathematics devoted to development of techniques to be employed in computation—*producing numbers.* In the present era of high-speed, relatively inexpensive digital computers this is an essential topic for engineers and scientists who must solve problems associated with predicting behavior of a broad spectrum of physical systems. While, in principle, one might expect to accomplish this via the usual software packages such as MatLab, it is crucial to understand how these packages work—what is actually doing the work inside them—so that if results are not as expected it is possible to understand the source of difficulty, and possibly fix it.

The goal of Section 001 of this course is to provide engineering and physics students with a broad overview of basic numerical analysis (*in a single semester!*) organized in such a way as to naturally culminate, toward the end of the semester, in an introduction to the numerical solution of partial differential equations, leading to the ability to solve a large percentage of problems likely to be encountered in research and practice. Despite this broad coverage of topics no previous mathematics background beyond sophomore calculus is presumed, but courses in analysis and/or differential equations are obviously helpful. Each topic will be introduced with a brief treatment of the underlying "pure" mathematics followed by *widely-used* appropriate numerical algorithms.

COURSE OUTLINE

I. Numerical Linear Algebra

    A. Solution of linear systems
        1. direct methods—Gaussian elimination, tridiagonl LU decomposition
        2. iterative methods—fixed-point iteration, Jacobi, Gauss-Seidel, SOR

    B. The algebraic eigenvalue problem—power method

II. Solution of Nonlinear Equations

    A. Fixed-point iteration, revisited
    B. Newton's method
    C. Newton's method for systems

III. Approximation Theory

    A. Approximation of functions
        1. nonlinear least squares—an application of Newton's method
        2. polynomial interpolation

    B. Numerical quadrature—approximation of definite integrals
        1. trapezoidal integration
        2. Richardson extrapolation

       3. Simpson's rule

  C. Finite-difference approximation of derivatives
       1. forward, backward, centered approximation of first derivatives
       2. higher-order derivative approximation

  D. Richardson extrapolation—in general

  E. Grid function convergence

IV. Ordinary Differential Equations

  A. Initial-value problems
       1. some basic mathematical observations
       2. forward and backward Euler methods
       3. trapezoidal integration
       4. explicit trapezoidal—a 2nd-order Runge-Kutta method
       5. stiff equations

  B. Boundary-value problems
       1. mathematical formulation
       2. finite-difference approximation
       3. treatment of boundary conditions
       4. solution of the linear algebra problem
       5. treatment of coordinate singularities
       6. quasilinearization of nonlinear problems—a function-space Newton's method
       7. introduction to the Galerkin procedure

V. Partial Differential Equations

  A. Overview of solution methods
  B. Classification of PDEs
  C. The heat equation
       1. basic mathematics
       2. forward Euler/centered approximation—stability considerations
       3. the Crank–Nicolson method
       4. Peaceman–Rachford ADI (if time permits)

  D. Laplace's equation
       1. problem formulation and discretization
       2. solution via successive overrelaxation (SOR)

  E. The wave equation
       1. mathematics of hyperbolic equations
       2. basic centered-difference approximation

HOMEWORK

There will be approximately **four** homework assignments (each consisting of several separate problems) throughout the semester, with due dates (somewhat) negotiable—but all students must submit solutions on the same date, and this date will precede the due date of any subsequent assignment. **All problems will involve writing and running high-level language (Fortran or C) computer codes**. Use of MatLab will <u>not</u> be permitted for

solving assigned problems, but it may be used for checking results produced by your codes. Copying codes from any source whatever (other students, other text books, *etc.*) is forbidden and will result in severe penalties—so, *just don't even think about it!*

Only <u>one</u> problem (to be selected randomly by me after the assignment has been turned in) from each set will be graded in detail, and the score from that problem will count 2/3 of the total score for the whole assignment. The remaining problems will be given equal weight to account for the remaining 1/3, and will be graded on a rather perfunctory basis.

Homework sets will not necessarily be equally weighted, but I will provide an indication of relative weighting for each set.

EXAMINATIONS

There will be <u>one</u> one-hour midterm given at approximately the middle of the semester (date to be determined later). The two-hour final exam *will be comprehensive*, but will emphasize material from the second half of the semester. Both exams will be closed book, closed notes, and neither will require use of calculators—hence, no calculators will be permitted.

GRADING

Grades will be assigned with appropriate "curving" as needed, but anyone accumulating 90% or more of the total points for the course is guaranteed a grade of **A**. Total course point assignment will be based on the following distribution:

Homework (total)	40%
Midterm	20%
Final Exam	40%

RECOMMENDED READING

E. Issacson and H. Keller, *Analysis of Numerical Methods*, Dover Pub. Co. (paperback)

J. M. McDonough, *Lectures in Basic Computational Numerical Analysis*, to be made available in PDF format from the UK Engineering website:

http://courses.engr.uky.edu/fall05/EGR/egr537-001

OFFICE HOURS

By appointment—send e-mail, or whenever you can catch me.