EE522 NOTES (5): ANTENNA ARRAY

Array: a set of antennas working together to produce certain radiation pattern.

Each antenna in an array is called an element antenna (or simply an element).

The elements in an array can be the same or different. In most practical cases, they are identical in construction (with different feedings).

Configuration of arrays: Linear (1D), planar (2D), and conformal (3D).

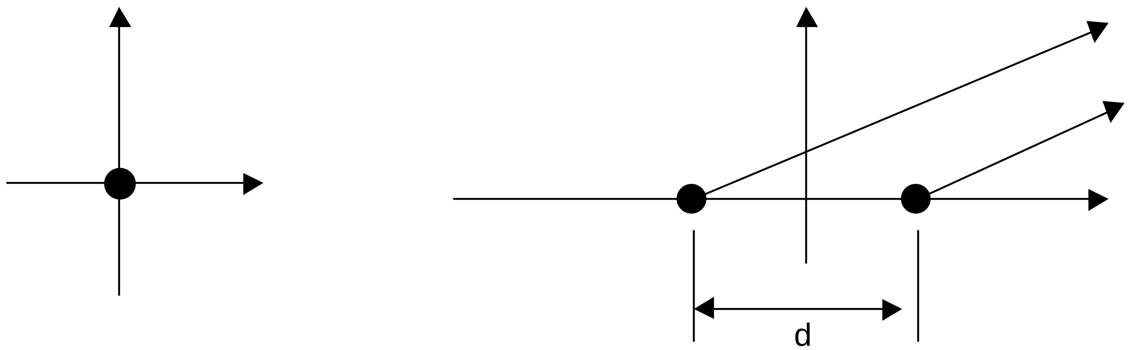
Array analysis: to obtain array factor (AF) given array configuration and element feedings.

Array synthesis: To determine the array configuration and/or element feedings to achieve desired array factor.

For array made up of identical elements,

(Array pattern) = (Array factor) X (element pattern)

Array analysis:



Consider an array of two identical elements. When the element is placed in the origin of the coordinate, the radiation is

$$\bar{E} = \hat{e}\, C\, I \frac{e^{-j\beta r}}{r} F_e(\theta,\phi)$$

If the two elements are placed in new positions, one at (0,0,d/2), and the other in (0,0,-d/2), and excited by I_1, and I_2, respectively, then the far-field from the elements are:

$$\bar{E}_1 = \hat{e}\, C\, I_1 \frac{e^{-j\beta r_1}}{r_1} F_e(\theta,\phi)$$

$$\bar{E}_2 = \hat{e}\, C\, I_2 \frac{e^{-j\beta r_2}}{r_2} F_e(\theta,\phi)$$

Using far-field approximation: $r_1 = r_2 = r$ for the amplitude factors, and

$$r_1 = r + \frac{d}{2}\cos\theta, \qquad r_2 = r - \frac{d}{2}\cos\theta$$

for the phase factors. The combined radiation is

$$\bar{E} = \hat{e}\,C\,I_1 \frac{e^{-j\beta(r+\frac{d}{2}\cos\theta)}}{r} F_e(\theta,\phi) + \hat{e}\,C\,I_2 \frac{e^{-j\beta(r-\frac{d}{2}\cos\theta)}}{r} F_e(\theta,\phi)$$

$$= \hat{e}\,C \frac{e^{-j\beta r}}{r} \left[I_1 e^{j(\beta d\cos\theta)/2} + I_2 e^{-j(\beta d\cos\theta)/2} \right] F_e(\theta,\phi)$$

Hence the radiation pattern is

$$F(\theta,\phi) = \left[I_1 e^{j(\beta d\cos\theta)/2} + I_2 e^{-j(\beta d\cos\theta)/2} \right] F_e(\theta,\phi) = \left(AF\right) \times F_e(\theta,\phi)$$

where the array factor (AF) is given by

$$\mathrm{AF} = I_1 e^{j(\beta d\cos\theta)/2} + I_2 e^{-j(\beta d\cos\theta)/2}$$

It depends on:

(1) the relative location of the elements
(2) the relative excitation of the elements.

Example: if the two elements are half-wave dipole, they are placed on z-axis and oriented parallel to z-axis. Find the array factor (AF) and the array pattern for the following cases:

(1) $I_1 = I_2 = 1, \quad d = \lambda/2$

(2) $I_1 = 1,\ I_2 = j, \quad d = 0.25\lambda$

Solution:

For both cases, the element pattern is

$$F_e(\theta) = \cos[(\pi/2)\cos\theta]/\sin\theta$$

(1) $\beta d/2 = \dfrac{2\pi}{\lambda}\dfrac{\lambda/2}{2} = \dfrac{\pi}{2},$

$$AF = e^{j(\pi/2)\cos\theta} + e^{-j(\pi/2)\cos\theta} = 2\cos[(\pi/2)\cos\theta]$$

The array pattern is

$$F(\theta,\phi) = \cos[(\pi/2)\cos\theta] \times \cos[(\pi/2)\cos\theta]/\sin\theta$$

$$= \cos^2[(\pi/2)\cos\theta]/\sin\theta$$

(2) $\beta d/2 = \pi/4$,

$$AF = e^{j(\pi/4)\cos\theta} + e^{j\pi/2 - j(\pi/4)\cos\theta} = e^{j(\pi/4)} 2\cos\left[(\pi/4)\cos\theta - \pi/4\right]$$

Array pattern is

$$F\left(\theta,\phi\right) = \cos\left[(\pi/4)\cos\theta - \pi/4\right] \times \cos[(\pi/2)\cos\theta]/\sin\theta$$

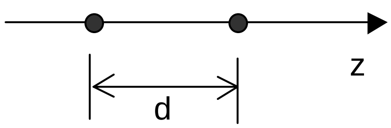
Example 2:

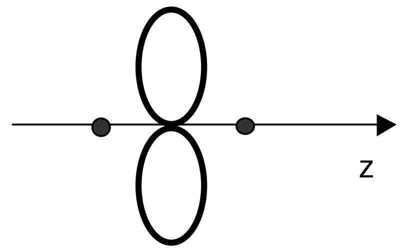
Two identical isotropic source

Equal excitation (for both amplitude and phase)

Separation: $\lambda / 2$

Location: z-axis.





(1) Sketch the polar radiation pattern by inspection

(2) Derive the exact array factor.

Solution:

(1) Since the excitations are the same,

$\theta = \pi / 2 \;\; or \;\; \theta = -\pi / 2$ have maximum radiation.

At $\theta = 0$, the phase difference from the source to the observation point is 180 (deg) out of phase because of half-wave distance. Hence at this angle, the radiation is 0. The same is true for $\theta = \pi$.

The radiation at other angles are between 0 and maximum. Hence we have the sketch as shown.

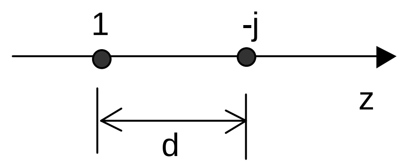
(2) $AF = e^{j\beta(d/2)\cos\theta} + e^{-j\beta(d/2)\cos\theta}$

Since $d = \lambda / 2, \;\; \beta d / 2 = (2\pi / \lambda)(\lambda / 2) = \pi / 2$

$$AF = \cos\left[(\pi / 2)\cos\theta\right]$$

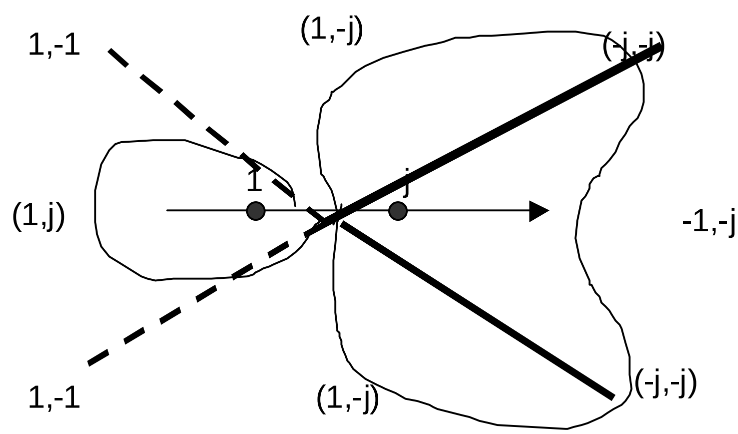
As expected, the radiation is 0 at $\theta = 0, \pi$, and is maximum for $\theta = \pm\pi / 2$.

Example 3:
Two identical isotropic source
Equal Amplitude
90 (deg) phase difference,
Separation: $\lambda/2$
Location: z-axis.



(1) Sketch the polar radiation pattern by inspection
(2) AF=?

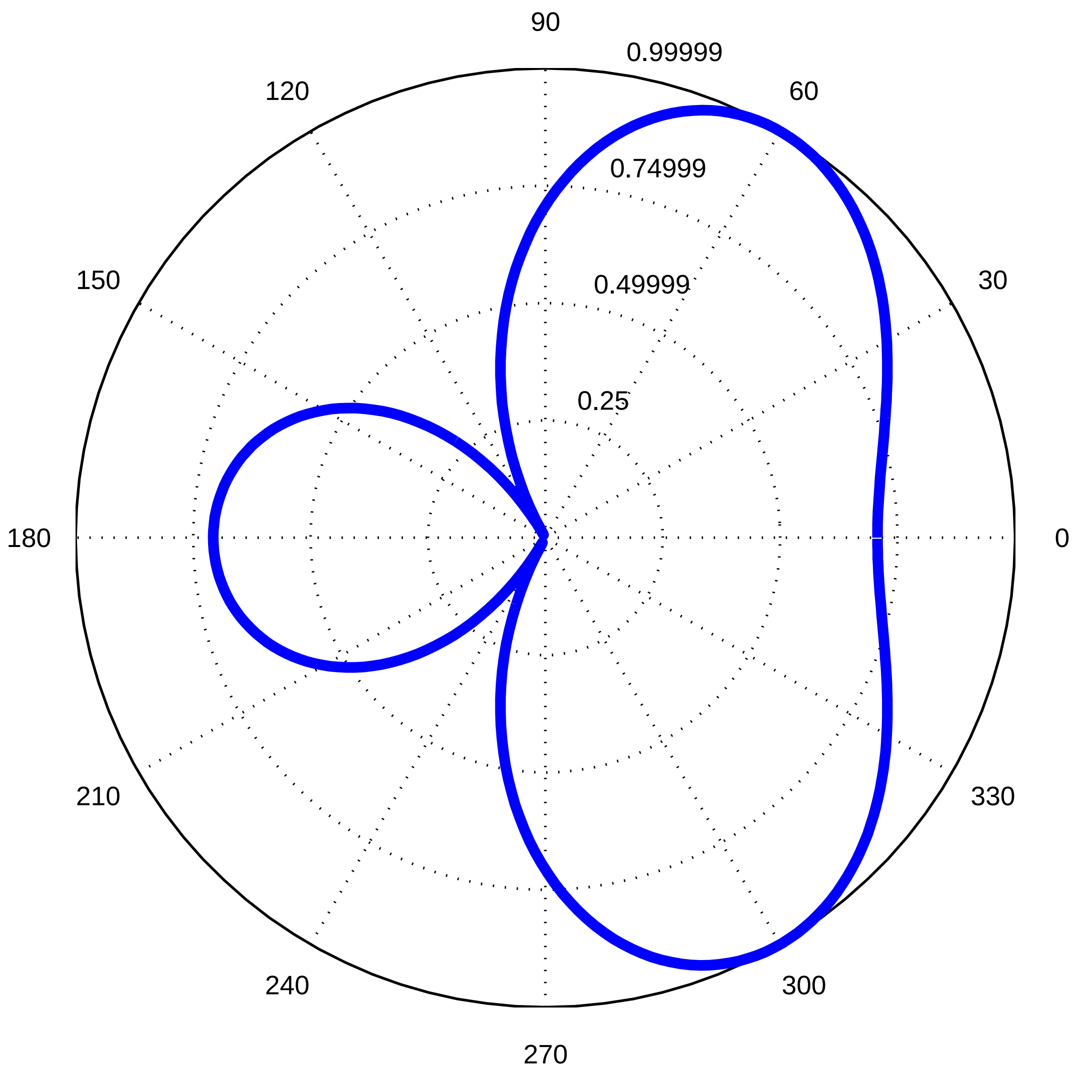
Solution: (1) As shown in the figure.



(2)

$$AF = I_1 e^{j\beta(d/2)\cos\theta - j\pi/2} + I_2 e^{j\beta(-d/2)\cos\theta}$$
$$= I_0 e^{-j\pi/4}\left[e^{j\beta(d/2)\cos\theta - j\pi/4} + e^{j\beta(-d/2)\cos\theta + j\pi/4}\right]$$
$$= e^{-j\pi/4} 2I_0 \cos\left[(\beta d/2)\cos\theta - \pi/4\right]$$
$$= e^{-j\pi/4} 2I_0 \cos\left[(\pi/2)\cos\theta - \pi/4\right]$$

The exact plot is shown on next page.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- **Array factor of linear arrays.**

For a linear array, the array factor can be obtained similarly as the two-element array case. Assuming an N-element array, the element excitations are: $I_0, I_1, \cdots, I_{N-1}$. If the location of the elements are $z_0, z_1, \cdots, z_{N-1}$, then the n-th element contribution to AF is given by $I_n e^{j\beta z_n \cos\theta}$, and the AF is the summation of these terms:

$$AF = I_0 e^{j\beta z_0 \cos\theta} + I_1 e^{j\beta z_1 \cos\theta} \cdots + I_{N-1} e^{j\beta z_{N-1} \cos\theta}$$

Equal space case:

$$z_0 = 0,\ z_1 = d,\ z_2 = 2d,\ \cdots, z_{N-1} = (N-1)d$$

$$AF = \sum_{n=0}^{N-1} I_n e^{j\beta nd\cos\theta} = \sum_{n=0}^{N-1} | I_n | e^{j(\beta nd\cos\theta + \alpha_n)}$$

Uniformly excited, equal spaced linear array with linear phase progression:

$$I_0 = 1, I_1 = e^{j\alpha}, I_2 = e^{j2\alpha}, \quad I_n = e^{jn\alpha}, \cdots$$

The array factor in this case have a closed form

$$AF = \sum_{n=0}^{N-1} e^{jn(\beta d\cos\theta + \alpha)} = \sum_{n=0}^{N-1} e^{jn\psi} = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}}$$

Reference:

$$S = 1 + x + x^2 + x^3 + \cdots + x^{N-1}$$

$$xS = \quad x + x^2 + x^3 + \cdots + x^{N-1} + x^N$$

$$(1-x)S = 1 - x^N \quad \Rightarrow S = (1 - x^N)/(1-x)$$

Consider

$$
\begin{aligned}
1-e^{jN\psi} &= e^{jN\psi/2}e^{-jN\psi/2} - e^{jN\psi/2}e^{jN\psi/2} \\
&= e^{jN\psi/2}\left(e^{-jN\psi/2} - e^{jN\psi/2}\right) \\
&= e^{jN\psi/2}\,2j\sin(N\psi/2)
\end{aligned}
$$

Ignoring the phase, the uniform excited (UE), equal spaced linear array (ESLA) has the array factor

$$AF = \frac{\sin(N\psi/2)}{N\sin(\psi/2)}, \qquad \text{UE, ESLA}$$

Observations:

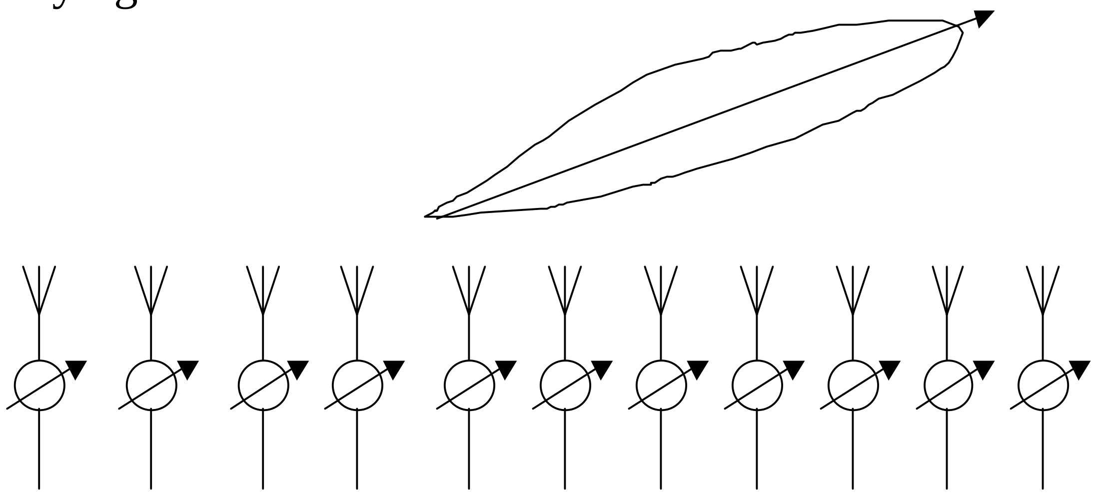
(1) Main lobe is in the direction so that

$$\psi = \beta d\cos\theta + \alpha = 0$$

(2) The main lobe narrows as N increases.
(3) Number of side lobes is N-1 in one period of $AF(\psi)$.
(4) SLL decreases as N increases.
(5) $AF(\psi)$ is symmetric about $\psi = \pi$.

- Electriconic beam scanning

The maximum radiation direction for UE-ESLA array is in direction θ_0 such that $\beta d \cos\theta_0 - \alpha = 0$. For fixed spacing (d), the main beam direction is controlled by the phase progression parameter α. Thus beam scan is realized by varying α.

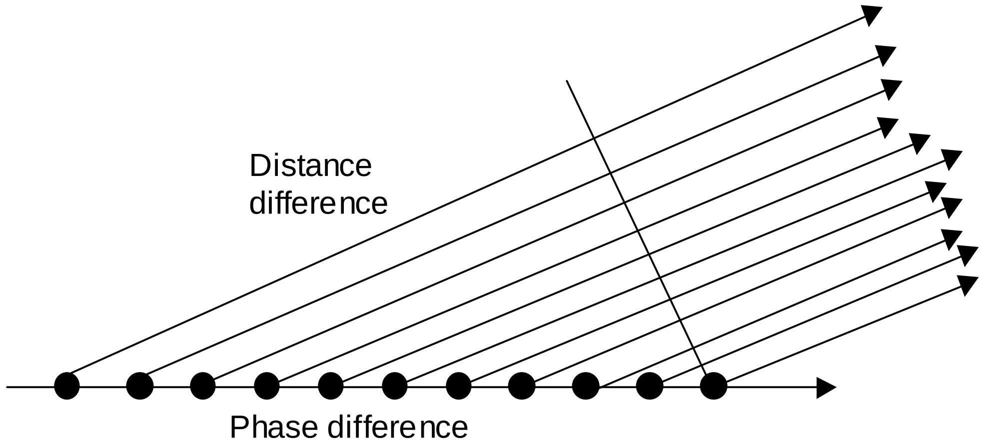


Pattern parameters for UE-ESLA
(for isotropic elements)

Half-Power beam width HP: When $Nd >> \lambda$,

$$HP = \begin{cases} 0.886 \dfrac{\lambda}{Nd} \csc\theta_0, & \text{Near Broadside} \\ 2\sqrt{0.886 \dfrac{\lambda}{Nd}}, & \text{Endfire} \end{cases}$$

Directivity: Need numerical integration.



Beam forming in an arbitrary direction is realized by compensation of the distance differences with the phase differences in the excitations.

Look at the sin(Nx)/(Nsinx) plot, there are two main beams in general (one at x=0, and the other at $x = 2\pi$). In many practical applications, a single pencil beam is desired.

How to select array parameters so that the array produces a single pencil beam?

Ordinary endfire array:

$$2\beta d \le 2\pi - \frac{\pi}{N}, \quad or \quad d \le \frac{\lambda}{2}\left[1 - \frac{1}{2N}\right]$$

Non-uniformly excited, equally spaced linear array (NE,ESLA)

Recall the AF for equally spaced linear array:

$$AF = I_0 + I_1 e^{j\beta d\cos\theta} \cdots + I_{N-1} e^{j\beta(n-1)d\cos\theta}$$

For linear phase progression, we define

$$I_n = A_n e^{jn\alpha}, \qquad \psi = \beta d\cos\theta + \alpha$$

$$AF = A_0 + A_1 e^{j\psi} + \cdots + A_{N-1} e^{j(N-1)\psi}$$

Denote $Z = e^{j\psi}$, then

$$AF = A_0 + A_1 Z + A_2 Z^2 + \cdots + A_{N-1} Z^{N-1}$$

Binomial distribution: $AF = (1+Z)^{N-1}$

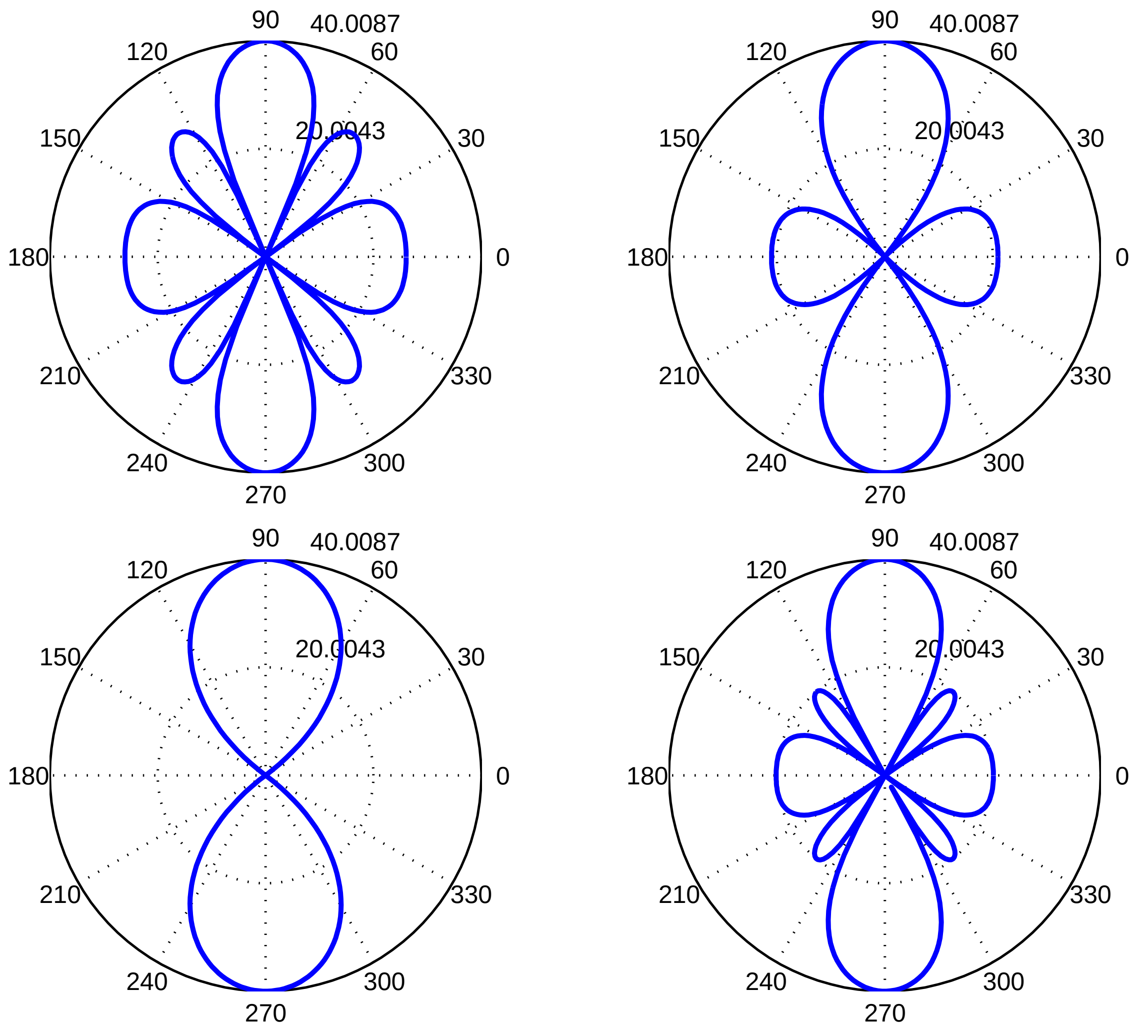
$$N = 2: \quad AF = 1 + Z$$

$$N = 3: \quad AF = 1 + 2Z + Z^2$$

$$N = 4: \quad AF = 1 + 3Z + 3Z^2 + Z^3$$

$$N = 5: \quad AF = 1 + 4Z + 6Z^2 + 4Z^3 + Z^4$$

The amplitude distribution is symmetric. The AF for binomial distribution has no sidelobes.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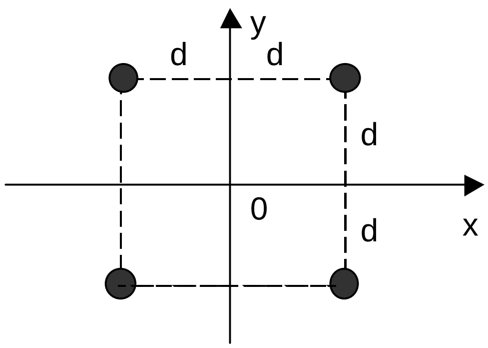
Two dimensional array

Example: Four isotropic elements are placed on the four corners of a rectangular region as shown below. Find the AR for this array (Assuming that the excitations are identical for the four elements).



Solution:

$$AF = I_1 e^{j\beta\hat{r}\cdot\overline{r_1}} + I_2 e^{j\beta\hat{r}\cdot\overline{r_2}} + I_3 e^{j\beta\hat{r}\cdot\overline{r_3}} + I_4 e^{j\beta\hat{r}\cdot\overline{r_4}}$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\overline{r_1} = \hat{x}d + \hat{y}d, \qquad \hat{r}\cdot\overline{r_1} = d\sin\theta\,(\cos\phi + \sin\phi)$$

$$\overline{r_2} = -\hat{x}d + \hat{y}d, \qquad \hat{r}\cdot\overline{r_2} = d\sin\theta\,(-\cos\phi + \sin\phi)$$

$$\overline{r_3} = -\hat{x}d - \hat{y}d, \qquad \hat{r}\cdot\overline{r_3} = d\sin\theta\,(-\cos\phi - \sin\phi)$$

$$\overline{r_4} = \hat{x}d - \hat{y}d, \qquad \hat{r}\cdot\overline{r_4} = d\sin\theta\,(\cos\phi - \sin\phi)$$

$$AF = e^{j\beta d\sin\theta(\cos\phi+\sin\phi)} + e^{j\beta d\sin\theta(\cos\phi-\sin\phi)}$$

$$+ e^{j\beta d\sin\theta(-\cos\phi-\sin\phi)} + e^{j\beta d\sin\theta(-\cos\phi+\sin\phi)}$$

$$= e^{j\beta d\sin\theta\cos\phi}\, 2\cos\left(\beta d\sin\theta\sin\phi\right) + e^{-j\beta d\sin\theta\cos\phi}\, 2\cos\left(\beta d\sin\theta\sin\phi\right)$$

$$= 2\cos\left(\beta d\sin\theta\sin\phi\right)\left[e^{j\beta d\sin\theta\cos\phi} + e^{-j\beta d\sin\theta\cos\phi}\right]$$

$$= 4\cos\left(\beta d\sin\theta\sin\phi\right)\cos\left(\beta d\sin\theta\cos\phi\right)$$

Generally, 2D rectangular arrays have the AF given by

$$AF(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} I_{mn} e^{j(\beta\hat{r}\cdot\bar{r}'_{mn}+\alpha_{mn})}$$

Where $\bar{r}'_{mn}$ is the location of the (m,n) element.

$$\bar{r}'_{mn} = \hat{x}x'_{mn} + \hat{y}y'_{mn} + \hat{z}z'_{mn}$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\beta\hat{r}\cdot\bar{r}'_{mn} = \beta\left(x'_{mn}\sin\theta\cos\phi + y'_{mn}\sin\theta\sin\phi + z'_{mn}\cos\theta\right)$$

Main bean pointing direction (θ_0,ϕ_0) is given by

$$\alpha_{mn} = -\beta\left(x'_{mn}\sin\theta_0\cos\phi_0 + y'_{mn}\sin\theta_0\sin\phi_0 + z'_{mn}\cos\theta_0\right)$$