Muffler Case Studies

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A Candidate Design Process

- Clarification of Task
- Conceptual Design
- Detailed Design
- Prototyping

Muffler Simulation

ISO/TR 11688-1 and Bockhoff, 2007
1. Clarification of Task

- Establish targets
  ✓ Sound pressure at a receiver location
  ✓ Transmission loss (include at least 10 dB slack)
  ✓ Insertion loss

- Keep records
  ✓ Source characteristics (sound pressure spectra, source impedance if possible)
  ✓ Flows
  ✓ Temperatures
  ✓ Insertion loss of mufflers
  ✓ Organize a database of past models
2. Conceptual Design

- Design rules
  - Cross flow mufflers are generally effective
  - Perforates are effective if there is flow
  - Avoid sharp edges in the flow

- Virtual Design
  - Plane wave modeling
  - Estimate from handbook equations
Plane Wave Modeling

Muffler Simulation

In course work

Source

A real muffler

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Noise and Vibration
Short Course
Plane Wave Modeling

Muffler Simulation

\[
\begin{align*}
\begin{bmatrix}
  p_1 \\
  S_1 u_1 \\
\end{bmatrix} &= [T]
\begin{bmatrix}
  p_2 \\
  S_2 u_2 \\
\end{bmatrix}
\end{align*}
\]
Plane Wave Modeling

Muffler Simulation

Straight Pipe

\[ L \]

Cone

\[ z_1 \]
\[ z_2 \]

Perforate

Helmholtz Resonator

\[ V \]
\[ S \]

Quarter Wave Tube

\[ S_B \]

\[ L_B \]
Plane Wave Quarter Wave Tubes

Muffler Simulation

Z_B

Z_B

Z_B
Plane Wave Modeling Tips

1. Examine the muffler to be modeled and determine a direction of sound propagation. This will normally be the direction having the longest cross-dimension and is also usually the same as the flow path.

2. Details like welds, seams, and fillets that are small compared to an acoustic wavelength can be ignored. However, leaks should be included.

3. Identify and include quarter wave tubes. Often in the case of right angle turns, quarter wave tubes are unintentionally included. Additionally, be sure to include mass effects in the case of extended inlets or outlets.
Plane Wave Modeling Tips

Muffler Simulation

4. The shape of the cross-section, whether circular, elliptical, or rectangular will have no effect.

5. Sound will be reflected due to even gradual changes in the cross-sectional area. Any noticeable area change should be accounted for. Gradual changes in cross-sectional area can be modeled as cones.

6. If the porosity (perforation rate) exceeds 30%, a perforate can generally be ignored and treated as open.
Example Small Engine Muffler

Muffler Simulation

Perforated

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Identify Flow Path
Identify Quarter Wave Tubes

Muffler Simulation
Transmission Loss

Muffler Simulation

Transmission Loss (dB)

Measurement
Sidlab

Frequency (Hz)
0 1000 2000 3000 4000
Example Cross-flow Muffler

Muffler Simulation

- Propagation
- Quarter-wavelength tube
- Cross-flow
- Helmholtz resonator
3. Detailed Design

Muffler Simulation

• Virtual prototyping using acoustic FEM and BEM
4. Prototyping

Muffler Simulation

- Construct muffler mock-up
  - Measure transmission loss in the lab to confirm modeling approach.
  - Measure insertion loss on product.
Example No Helmholtz Resonator

Muffler Simulation

Transmission Loss (dB)

Frequency (Hz)

- Measurement
- SIDLAB
- MAP
Example Helmholtz Resonator Included
Summary

Muffler Simulation

- Proposed design process for mufflers.
- Demonstrated process on development a cross-flow muffler.
Additional Notes

Muffler Simulation

- Transmission Loss Simulation
- Perforates
Simulation

Muffler Simulation

- Acoustic FEM (Sysnoise)
- Both 3-Point and 4-Pole approaches
Modeling Strategy

Muffler Simulation

\[
\begin{align*}
\begin{bmatrix} p_1 \\ S_1 v_1 \end{bmatrix} &= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ S_2 v_2 \end{bmatrix} \\
\end{align*}
\]

3-D Waves

Plane Waves
Calculate Four Poles Using BEM

\[ \begin{align*}
\{p_2\} &= \begin{bmatrix} A^* & B^* \end{bmatrix} \begin{bmatrix} u_2 \end{bmatrix} \\
\{p_1\} &= \begin{bmatrix} C^* & D^* \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}
\end{align*} \]

**Improved Four Pole Method**

1st Run

\[
A^* = p_2 \bigg|_{u_2=1, u_1=0}
\]

\[
C^* = p_1 \bigg|_{u_2=1, u_1=0}
\]

2nd Run

\[
B^* = p_2 \bigg|_{u_2=0, u_1=-1}
\]

\[
D^* = p_2 \bigg|_{u_2=0, u_1=-1}
\]
Calculate Four Poles Using BEM

\[
\begin{aligned}
\left\{ \begin{array}{c}
p_1 \\ S_1v_1
\end{array} \right\} &= \left[ \begin{array}{cc}
A & B \\
C & D
\end{array} \right] \left\{ \begin{array}{c}
p_2 \\ S_2v_2
\end{array} \right\} \\
A &= A^* / C^* \\
B &= \frac{1}{S_2} \left( B^* - A^* D^* / C^* \right) \\
C &= S_1 / C^* \\
D &= -\left( \frac{S_1}{S_2} \right) D^* / C^*
\end{aligned}
\]

\[
TL = 20 \log_{10} \left\{ \frac{1}{2} \left[ A + S_2 \frac{B}{\rho c} + C \frac{\rho c}{S_1} + \frac{S_2}{S_1} D \right] \right\} + 10 \log_{10} \left( \frac{S_1}{S_2} \right)
\]
TL Comparison Muffler 1

Muffler Simulation

Frequency (Hz)

TL (dB)

Two Load Measurement

FE Simulation

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TL Comparison Muffler 2

Muffler Simulation

TL (dB)

Frequency (Hz)

Two Load Measurement
FE Simulation
Additional Notes

Muffler Simulation

- Transmission Loss Simulation
- Perforates
Transfer Impedance

Muffler Simulation

Perforated plate

\[ Z_{tr} = \frac{P_1 - P_2}{v_n} \]
Transfer Impedance Measurement

Muffler Simulation

\[ Z_{tr} = \frac{p_1 - p_2}{u} = Z_1 - Z_2 \]

Wu, 2003
Empirical Equations

Muffler Simulation

Eq. 1 (Sullivan and Crocker, 1978)
\[ Z_{tr} = \frac{6 \times 10^{-3} + jk(t + 0.75d_h)}{\sigma} \]

Eq. 2 (Sullivan and Crocker, 1978)
\[ Z_{tr} = \frac{2.4 + j0.02f}{\rho c \cdot \sigma} \]

Eq. 3 (Rao and Munjal, 1986)
\[ Z_{tr} = \frac{7.337 \times 10^{-3} (1 + 72.23M) + j2.2245 \times 10^{-5} f (1 + 51t)(1 + 204d_h)}{\sigma} \]
Model Based Equation

Equation 4 (Coelho, 1984)

\[ Z_{tr} = \frac{\rho (d' / d_h) \sqrt{8v \omega} + (\rho / 8c)(\omega d_h)^2}{\sigma \cdot \rho c} + j(\omega \rho)[d'' + (d' / d_h) \sqrt{8v / \omega}] \]

where \( \nu \) is the gas kinematic viscosity and

\[ d' = t + d_h, \quad d'' = t + (8 / 3\pi)d_h(1 - 0.7\sqrt{\sigma}) \]

\[ \omega = 2\pi f \]
Implementation in Virtual.Lab

Muffler Simulation

\[
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_4 & \alpha_5
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix}
+ 
\begin{bmatrix}
\alpha_3 \\
\alpha_6
\end{bmatrix}
\]

or,

\[
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_4 & \alpha_5
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

where \( \beta \) is the transfer admittance:

\[
\beta = \frac{1}{Z_{tr}}
\]
Perforated Tube Test Cases

Muffler Simulation

Perforated tube

Units: mm
# Perforated Tubes

## Muffler Simulation

<table>
<thead>
<tr>
<th>Name</th>
<th>Thickness ( t ) (mm)</th>
<th>Hole Dia. ( d ) (mm)</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube 1</td>
<td>0.3</td>
<td>3.2</td>
<td>9.8%</td>
</tr>
<tr>
<td>Tube 2</td>
<td>0.3</td>
<td>3.2</td>
<td>5.8%</td>
</tr>
<tr>
<td>Tube 3</td>
<td>0.3</td>
<td>4.8</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

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TL Test Setup

Muffler Simulation
Tube 1 (Porosity = 9.8 %)

Muffler Simulation

- SYSNOISE (Eq. 1)
- Measured

Frequency (Hz)

TL (dB)
Tube 2 (Porosity = 5.8 %)

Muffler Simulation

TL (dB)

SYSNOISE (Eq. 1)
Measured

Frequency (Hz)
Tube 3 (Porosity = 5.8 %)

Muffler Simulation

SYSNOISE (Eq. 1)

Measured

TL (dB)

Frequency (Hz)

SYSNOISE (Eq. 1)

Measured