Chapter 10 – Sound in Ducts

Types of Mufflers
1. Dissipative (absorptive) silencer:
   - Sound is attenuated due to absorption (conversion to heat)
   - Duct or pipe
   - Sound absorbing material (e.g., duct liner)

2. Reactive muffler:
   - Sound is attenuated by reflection and "cancellation" of sound waves
   - Compressor discharge details
   - 40 mm

3. Combination reactive and dissipative muffler:
   - Sound is attenuated by reflection and "cancellation" of sound waves + absorption of sound
   - Sound absorbing material
   - Perforated tubes

Performance Measures Transmission Loss
Transmission loss (TL) of the muffler:

\[ TL(\text{dB}) = 10 \log_{10} \frac{W_1}{W_2} \]

Performance Measures Insertion Loss
Insertion loss (IL) in dB:

\[ IL(\text{dB}) = \text{SPL}_1 - \text{SPL}_2 \]

Insertion loss depends on:
- TL of muffler
- Lengths of pipes
- Termination (baffled vs. unbaffled)
- Source impedance

Note: TL is a property of the muffler; IL is a "system" performance measure.
Example TL and IL

Acoustic System Components

Summary 1

- Dissipative mufflers attenuate sound by converting sound energy to heat via viscosity and flow resistance – this process is called sound absorption.
- Common sound absorbing mechanisms used in dissipative mufflers are porous or fibrous materials or perforated tubes.
- Reactive mufflers attenuate sound by reflecting a portion of the incident sound waves back toward the source. This process is frequency selective and may result in unwanted resonances.
- Impedance concepts may be used to interpret reactive muffler behavior.

The Helmholtz Resonator

Named for:
Hermann von Helmholtz, 1821-1894, German physicist, physician, anatomist, and physiologist.

Major work: Book, On the Sensations of Tone as a Physiological Basis for the Theory of Music, 1862.

Helmholtz Resonator Example

A 12-oz (355 ml) bottle has a 2 cm diameter neck that is 8 cm long. What is the resonance frequency?

\[ f_r = \frac{c}{2\pi} \sqrt{\frac{S_B}{L'}} \times \frac{\pi (0.02)^2}{4} \]

\[ f_r = 182 \text{ Hz} \]
**Helmholtz Resonator as a Side Branch**

\[ |z_B| = \frac{P}{S_B u_B} = \frac{1}{\omega M - \frac{1}{\omega^2 K}} \]

*Anechoic termination*

- \( V = 0.001 \text{ m}^3 \)
- \( L = 2.5 \text{ mm} \)
- \( S_B = 2 \times 10^{-5} \text{ m}^2 \)
- \( S = 8 \times 10^{-4} \text{ m}^2 \)
- \( f_n = 154 \text{ Hz} \)

**Network Interpretation**

\[ z_B = \frac{P}{S_B u_B} = \frac{1}{\omega M - \frac{1}{\omega^2 K}} \]

*Can we make \( z_B \) zero?*

\[ z_B \rightarrow 0 \quad \text{when} \quad \omega = \frac{1}{\sqrt{s K}} \]

*(Produces a short circuit and \( P \) is theoretically zero.)*

**A Tuned Dynamic Absorber**

\[ M_2 / M_1 = 0.5 \]

\[ K_2 M_2 = K_1 M_1 \]

**Resonances in an Open Pipe**

\( \lambda_1 = \frac{2 L}{\pi} \rightarrow f_1 = \frac{c}{\lambda_1} = \frac{340}{6.75} \approx 50.75 \text{ Hz} \)

\( \lambda_2 = \frac{2 L}{\pi} \rightarrow f_2 = \frac{340}{13.5} \approx 25.53 \text{ Hz} \)

*etc.*

**Example – HR Used as a Side Branch**

\[ |z_B| = \frac{P}{S_B u_B} = \frac{1}{\omega M - \frac{1}{\omega^2 K}} \]

*Anechoic termination*

- \( V = 750 \text{ cm}^3 \)
- \( L = 2.5 \text{ cm} (L' = 6.75 \text{ cm}) \)
- \( D = 5 \text{ cm} (D' = 19.6 \text{ cm}) \)
- \( D' = 10 \text{ cm} (S = 78.5 \text{ cm}^2) \)
- \( f_n = 340 \text{ Hz} \)

*E.g., engine intake systems*
The Quarter-Wave Resonator

The Quarter-Wave Resonator has an effect similar to the Helmholtz Resonator:

\[ \frac{S}{S_B} = \frac{1}{2} \left( \frac{\sin \frac{n\pi}{L}}{ \frac{n\pi}{L} } \right) \]

where \( n = 1, 3, 5, \ldots \)

\[ c_B = \frac{\omega}{L} \cos \left( \frac{n\pi}{L} \right) \]

\[ c_o = \frac{\omega}{L} \sin \left( \frac{n\pi}{L} \right) \]

\[ f_n = \frac{nc}{4L} \]

\[ TL = 10 \log \left( \tan \left( \frac{\omega c}{2L} \right) + \frac{4S}{S_B} \right) \]

\[ z_B = -\frac{\rho c_S}{S_B} \cos \left( \frac{n\pi}{L} \right) \]

where \( \omega = \sqrt{\frac{k}{m}} \) and \( m = \frac{A_C}{A_T} \)

Summary 2

- The side-branch resonator is analogous to the tuned dynamic absorber.
- Resonators used as side branches attenuate sound in the main duct or pipe.
- The transmission loss is confined over a relatively narrow band of frequencies centered at the natural frequency of the resonator.

Quarter Wave Tube + Helmholtz Resonator

Extended Inlet Muffler

\[ Z = \frac{T}{180} \]
Combining Component Transfer Matrices

\[
\begin{bmatrix}
\mathbf{p}_1 \\
\mathbf{u}_1
\end{bmatrix} =
\begin{bmatrix} A & B \\ C & D \end{bmatrix}
\begin{bmatrix}
\mathbf{p}_2 \\
\mathbf{u}_2
\end{bmatrix}
\]

Transfer matrix of \(p\) component

\[
\begin{bmatrix}
\mathbf{p}_1 \\
\mathbf{u}_1
\end{bmatrix} =
\begin{bmatrix} A & B \\ C & D \end{bmatrix}
\begin{bmatrix}
\mathbf{p}_2 \\
\mathbf{u}_2
\end{bmatrix}
\]

Transfer, transmission, or four-pole matrix

(\(A, B, C,\) and \(D\) depend on the component)
Expansion Chamber Muffler

Transfer Matrix of a Side Branch

Helmholtz Resonator Model

Performance Measures Transmission Loss

\[
\begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix}
\]

Transmission loss (TL) of the muffler:

\[
TL (\text{dB}) = 10 \log_{10} \left( \frac{W_0}{W_f} \right)
\]

\[
TL = 10 \log_{10} \left( \frac{S_1}{S_2} \frac{S_3 - B}{S_3} \frac{\rho c}{S_2} \frac{S_4}{S_5} \frac{S_6 - D}{S_6} \right)
\]
Resonances can form in the exhaust and tail pipes as well as within the muffler.

Transfer Impedance

Source Impedance

Source/Load Concept
Insertion Loss Prediction

Source Impedance Series Impedance

Source Impedance Parallel Impedance

Derivation Insertion Loss

Derivation Insertion Loss
Summary 3

- The transfer matrix method is based on plane wave (1-D) acoustic behavior (at component junctions).
- The transfer matrix method can be used to determine the system behavior from component "transfer matrices."
- Applicability is limited to cascaded (series) components and simple branch components (not applicable to successive branching and parallel systems).