Phasor Circuit Analysis

Phasor Diagrams, Voltage and Current Division

Phasor Diagram

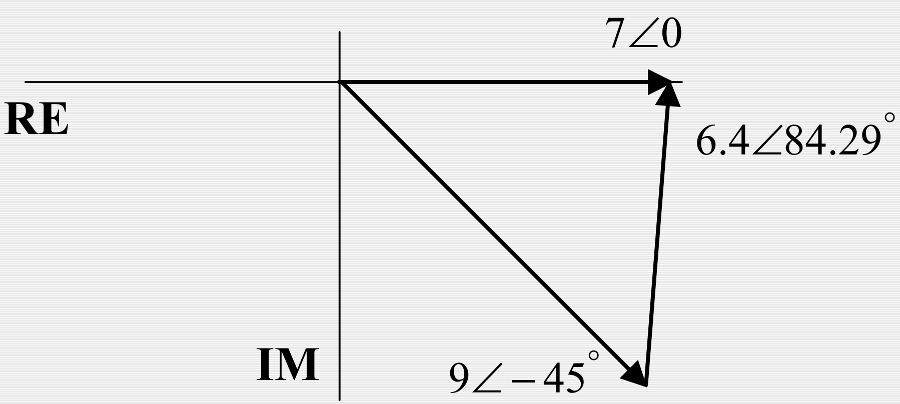
Phasors are denoted by vectors in 2-D space. Phasor diagrams graphically illustrate the summation of complex values

Consider the following summation of complex numbers resulting from setting up a KVL or KCL equation:

$$7\angle 0 = 9\angle -45^{\circ} + 6.4\angle 84.29^{\circ}$$

$$7 = 9\cos(-\pi\, 45/180) + j9\sin(-\pi\, 45/180) + 6.4\cos(\pi\, 84.29/180) + j6.4\sin(\pi\, 84.29/180)$$

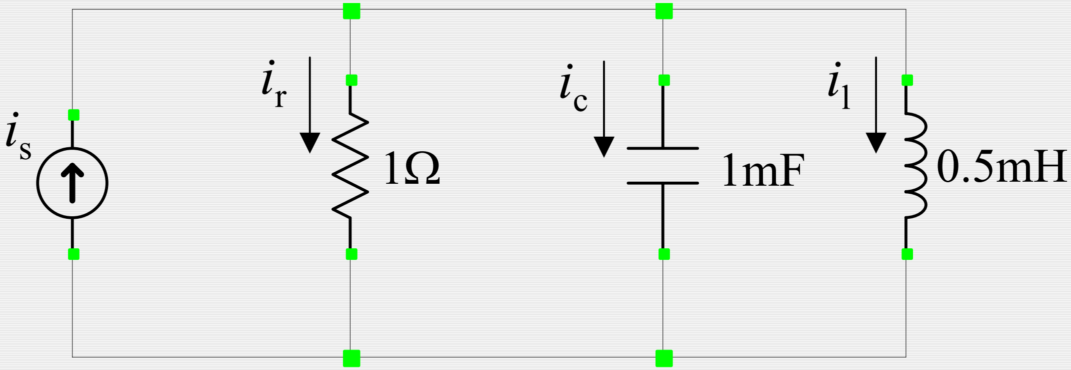
$$7 = 6.36 - j6.36 + 0.64 + j6.36$$



Phasor Diagram Example

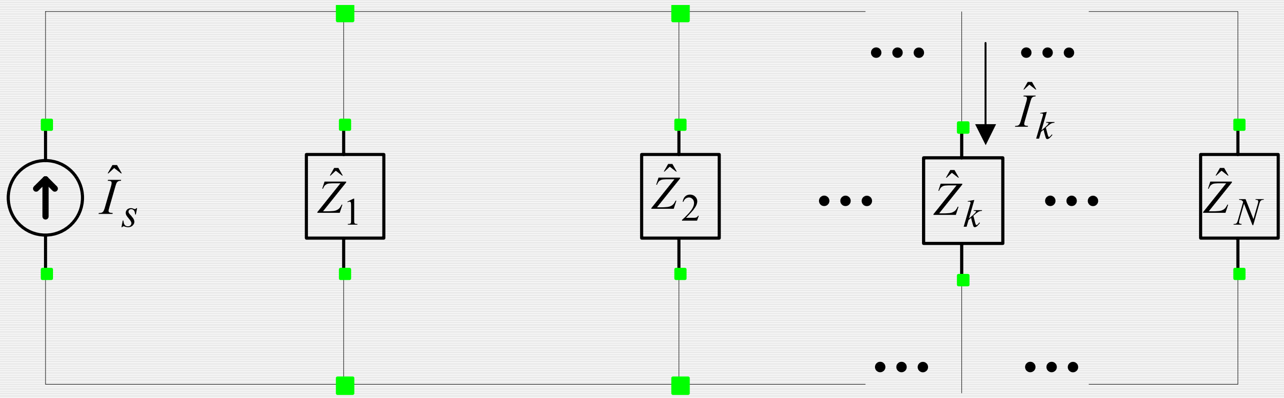
Draw Currents i_s, i_r, i_l, i_c in a phasor diagram to show $i_s = i_r + i_l + i_c$

where $i_s(t) = \sin(1000t)$ A



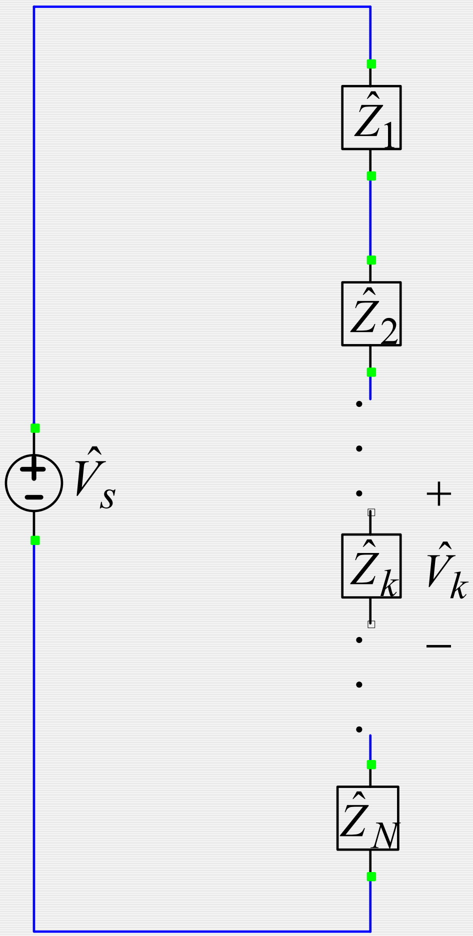
Current Division

By substituting impedance in for resistance (or admittance for conductance), the current division formula can be generalize to:



$$\hat{I}_k = \hat{I}_s \frac{\frac{1}{\hat{Z}_k}}{\frac{1}{\hat{Z}_1} + \frac{1}{\hat{Z}_2} + \cdots + \frac{1}{\hat{Z}_k} + \cdots + \frac{1}{\hat{Z}_N}} = \hat{I}_s \frac{\hat{Y}_k}{\hat{Y}_1 + \hat{Y}_2 + \cdots + \hat{Y}_k + \cdots + \hat{Y}_N}$$

Voltage Division



By substituting impedance in for resistance, the voltage division formula can be generalize to:

$$\hat{V}_k = \hat{V}_s \frac{\hat{Z}_k}{\hat{Z}_1 + \hat{Z}_2 + \cdots + \hat{Z}_k + \cdots + \hat{Z}_N}$$

In-Phase and Out-of-Phase

Voltages and currents are considered *in phase* if the phase angle between their phasor quantities is zero, otherwise they are *out of phase* by and amount equal to their phase difference.

Which quantities are in phase?

$\hat{V}_1\angle 45°$ $\quad \hat{V}_2\angle 0°$ $\quad \hat{V}_2\angle -90°$ $\quad \hat{I}_1\angle 270°$ $\quad \hat{I}_2\angle -45°$ $\quad \hat{I}_2\angle 45°$

Phase Lead and Phase Lab

Out of phase voltages and currents are considered to either lead or lag each other. If the phase of quantity 1 is subtracted from the phase of quantity 2 and the result is positive, then quantity 1 leads quantity 2 or equivalently quantity 2 lags quantity 1.

Given

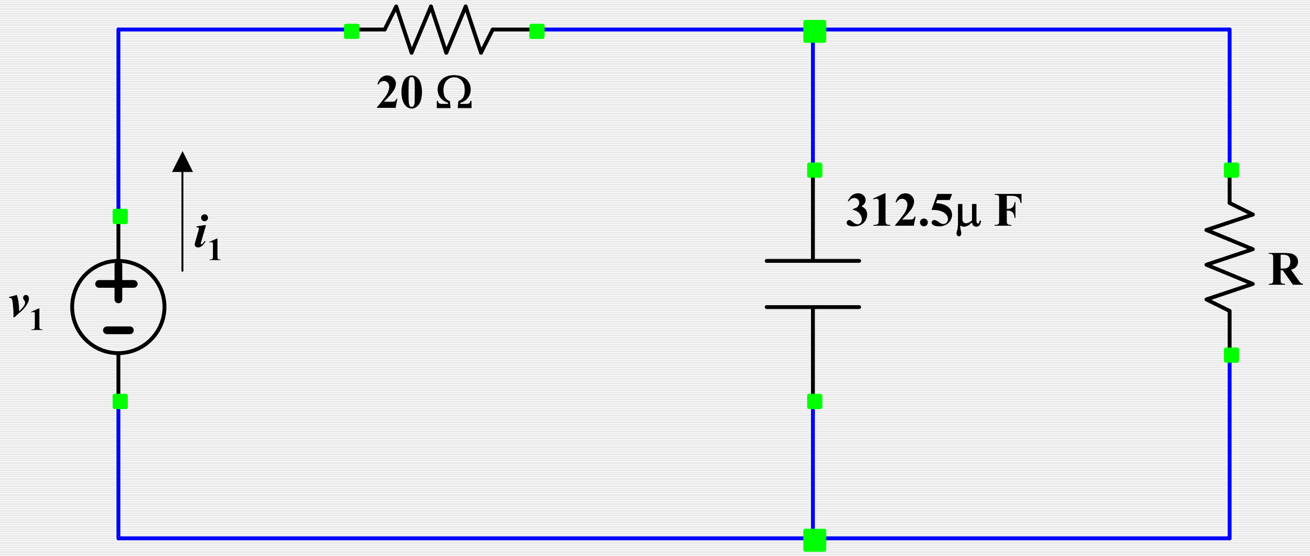
$\hat{V}_1\angle 45°$ $\hat{V}_2\angle 0°$ $\hat{V}_3\angle -90°$ $\hat{I}_1\angle 270°$ $\hat{I}_2\angle -45°$ $\hat{I}_3\angle 45°$

Describe the phase difference between

- v_1 and i_1
- v_2 and i_2
- v_3 and i_3

Eample

Find R such that the v_1 leads i_1 by 15 degrees, if $v_1 = \cos(400t)$ V



Show R = 15 Ω