Phasor Analysis

Phasors, Impedance, SPICE, and Circuit Analysis

Impedance

The conversion of resistive, inductive, and capacitive elements to impedance for a sinusoidal excitation at frequency ω is given by:

$$X_L = \omega L \quad \text{(Reactance)}$$

$$X_C = -\frac{1}{\omega C} \quad \text{(Reactance)}$$

$$R = R \quad \text{(Resistance)}$$

In general impedance is a complex quantity with a resistive component (real) and a reactive component (imaginary):

$$\hat{Z} = R + jX = \sqrt{R^2 + X^2} \angle \tan^{-1}\left(\frac{X}{R}\right)$$

Phasors

Sources can be converted to phasor notation as follows:

$$A\cos(\omega t+\theta) \Leftrightarrow A\angle\theta \qquad\qquad A\sin(\omega t+\theta) \Leftrightarrow A\angle\theta - 90^{\circ}$$

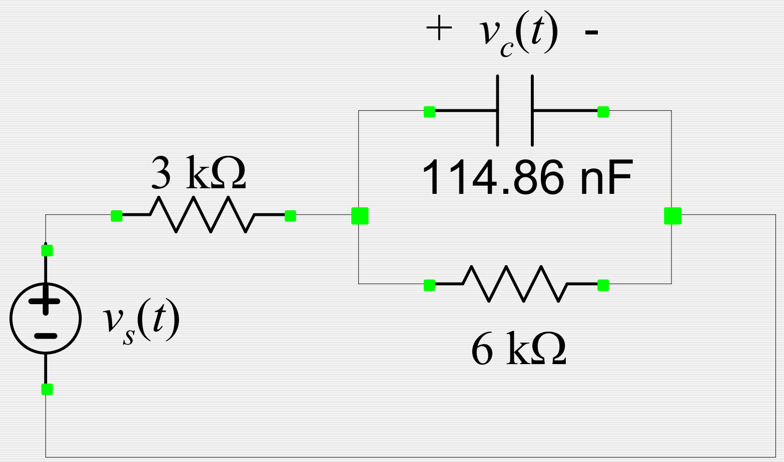
This can be applied to all sources of the same frequency, where ω is used in the impedance conversion of the circuit.

If sources of different frequencies exist, superposition must applied to solve for a given voltage or current:

1. **Select sources with a common ω and deactive all other sources.**
2. **Convert circuit elements to impedances.**
3. **Solve for desired voltage or current for selected ω.**
4. **Repeat steps 1 through 3 for new ω until all sources have been applied.**
5. **Add together all time-domain solutions solutions obtain in Step 3.**

Loop Analysis Example

Determine the steady-state response for $v_c(t)$ when $v_s(t) = 5\cos(800\pi t)$ V

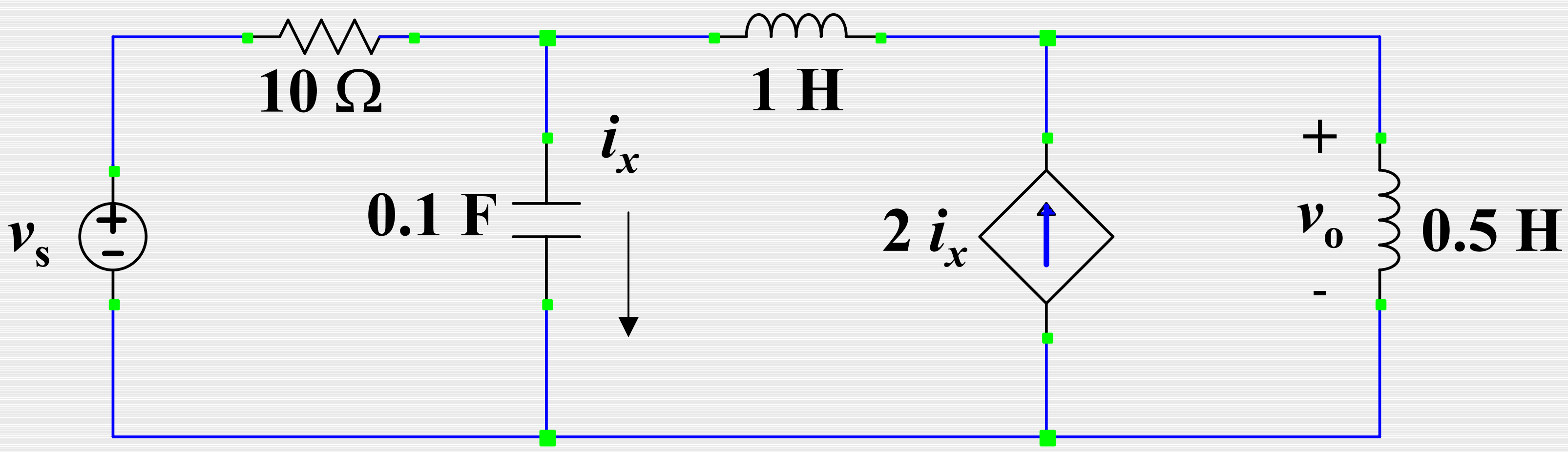


Show:

$$\hat{V}_C = 2.5000 - j1.4434 = 2.8868\angle -30^\circ \Leftrightarrow v_C(t) = 2.8868\cos\left(800\pi t - \frac{\pi}{6}\right) \text{ V}$$

Nodal Analysis Example

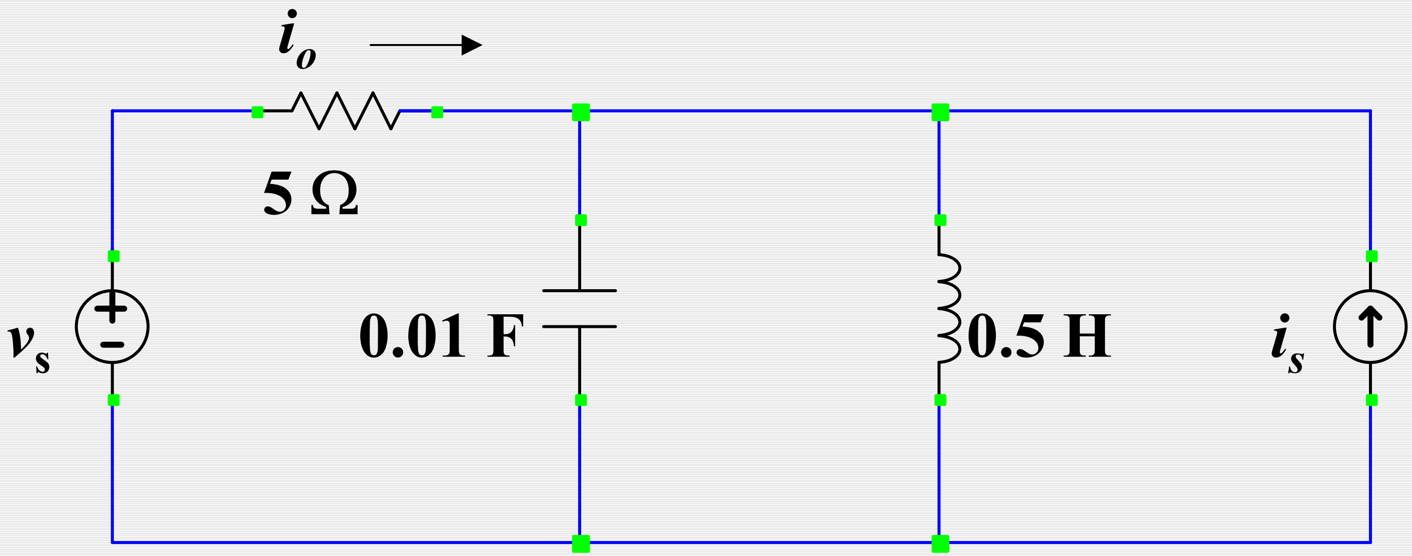
Find the steady-state value of $v_o(t)$ in the circuit below, if $v_s(t) = 20\cos(4t)$:



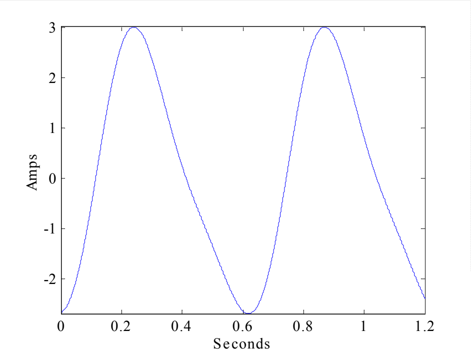
Show: $v_0(t) = 13.91\cos(4t + 198.3°)$

Multiple Source Example

Find i_o if $i_s = 3\cos(10t)$ and $v_s = 6\cos(20t + 60°)$

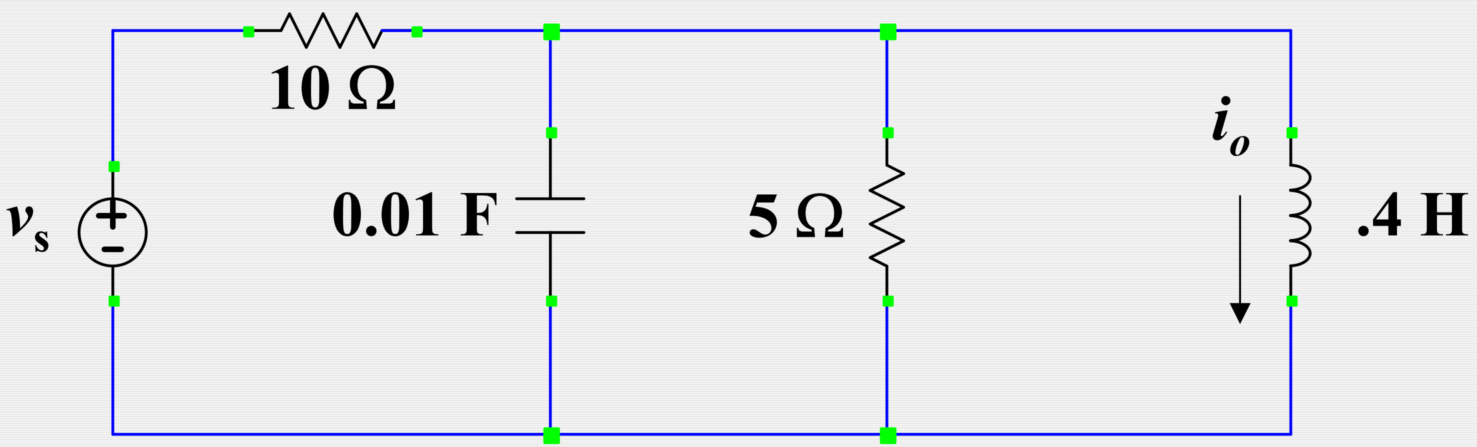


Show $i_o = 0.54\cos(20t + 123.4°) + 2.7\cos(10t - 153.4°)$



Equivalent Circuit Example

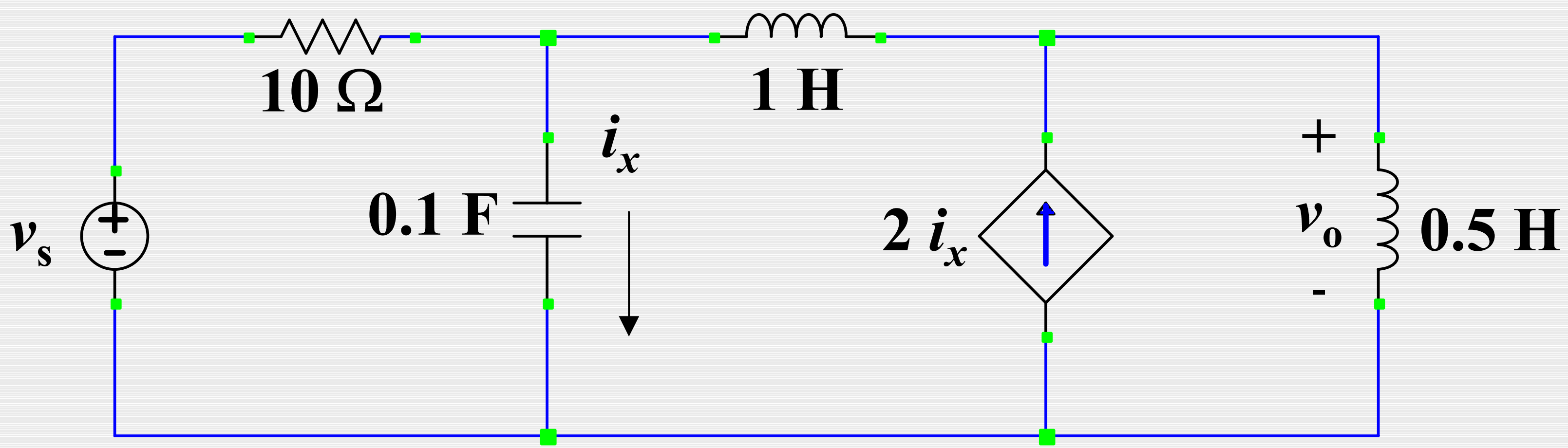
Find i_o steady-state using Norton's Theorem, if $v_s(t) = 2\sin(10t)$:



Show $i_s(t) = .2\sin(10t)$; $Z_{th} = 3\text{-}j = 3.2\angle\text{-}18.4^{o}$;
$i_o = 0.15\cos(10t\text{-}153.4^{o})$

Equivalent Circuit Example

Find v_o steady-state using Thévenin's Theorem, if $v_s(t) = 20\cos(4t)$:



$\hat{I}_{sc} = 6.1\angle 33.7°$ $\hat{Z}_{th} = 1.75\angle 137.7°$

$\hat{V}_{oc} = 10.67\angle -104°$ $v_o(t) = 13.04\cos(4t - 161.56°)$