Pulse Response - First Order Circuits

Unit Step functions, Pulse Sources, and SPICE

The Unit Step Function

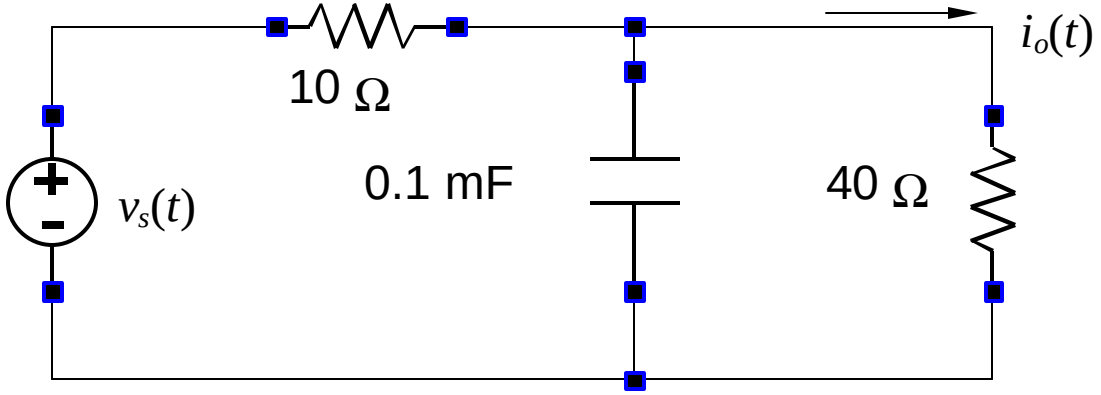
The unit step function models the behavior of a switch (i.e. off-on) and can be used to describe more complex circuit sources.

$$u(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ 1 & \text{for} \quad t > 0 \end{cases}$$

- Multiplying $u(t)$ by a constant changes the value of $u(t)$ for $t > 0$.
- Subtracting a positive number from the argument of $u(t)$ shifts the step in the positive t direction.
- Adding step functions together results in piece-wise steps with the resultant values equal to the sum of step values in each t interval.

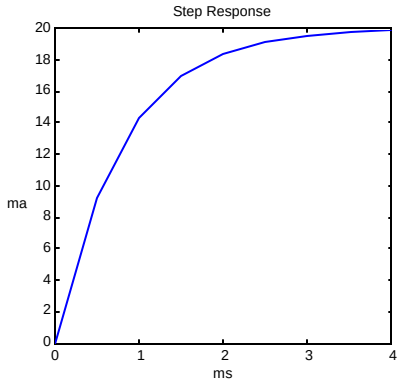
Example

Determine the unit step response for $i_o(t)$ the circuit below with input $v_s(t) = u(t)$:



Show:

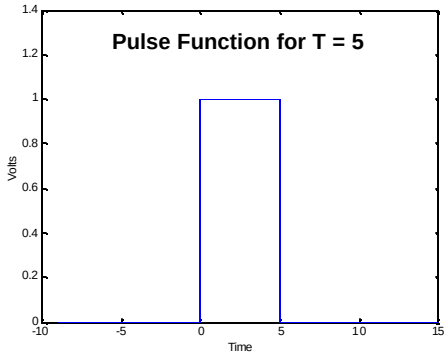
$$i_{ostep}(t) = \frac{\left(0.8 - 0.8\exp(-1250\,t)\right)u(t)}{40} \text{ amps}$$



The Unit Pulse Function

The unit pulse function is defined as :

$$v(t;T)=\begin{cases}0 & \text{for} \quad t<0\\ 1 & \text{for} \quad 0<t<T\\ 0 & \text{for} \quad t>T\end{cases}$$

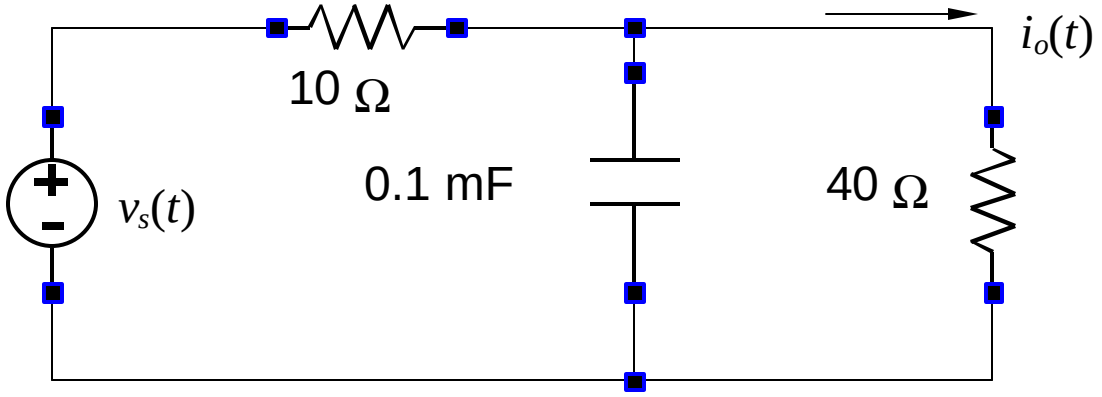


It can be expressed in terms of a combination of unit step functions:

$$v(t)==\left[u\left(t\right)-u\left(t-T\right)\right]$$

Example

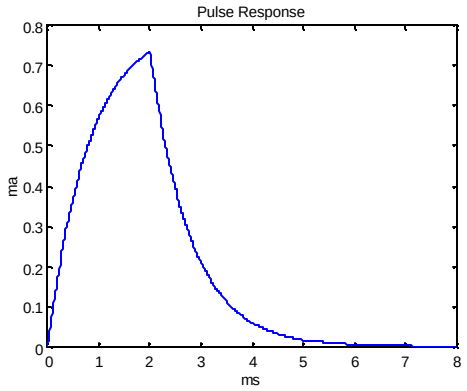
Determine the pulse response for $i_o(t)$ the circuit below with input $v_s(t) = 40v(t)$ V, where pulse duration T is 2 ms:



Show:

$$i_{0\text{pulse}}(t) = \left[\left(0.8 - 0.8\exp(-1250t)\right)u(t) - \left(0.8 - 0.8\exp(-1250(t-2\text{m}))\right)u(t-2\text{m})\right]\text{amps}$$

$$i_{o\text{pulse}}(t) = 0.8\left[u(t) - u(t-2\text{m})\right] - 0.8\exp(-1250t)\left[u(t) - 12.1825u(t-2\text{m})\right]\text{amps}$$



Plotting Transient Responses

- Programs written for Matlab can be created either as functions or scripts. For plotting transient responses it is convenient to write a function that acts like the unit step.
- A function acts like a subroutine in the Matlab workspace. Arguments must be passed to it and it assigns its output to variables in the workspace. Intermediate variable use by the function are cleared.

Creating a Function

- A function is a text file (can use Notepad or Matlab's editor. If using a word processor, you must store the as a text file.)
- The first line begins with the word 'function' and is followed by the syntax of the function you wish to create.
- The following lines can be comments, other Matlab functions, and commands to process the input arguments to create the output arguments.
- The file must be saved with a based name the same as the function and a '.m' extension in the current or working directory.

Example

- Create a function called ‘unit’ such that for an input array it will output a corresponding array of zeros and ones as the unit step function would do. Must save as text file called unit.m

```
function    out = unit(t)
%  This function is the unit step:
%
%   sig = unit(t), where SIG and T are vectors of the same dimension.
%
%   elements of out are either 0 or 1 corresponding to elements of T
%   being negative or non-negative, respectively.
%
%       written by Kevin D. Donohue   2/93
out = zeros(size(t));            %  Initialize output array to zeros
ind_of_positive = find(t >= 0);   %  Find index array of positive values on t-axis
out(ind_of_positive) = 1;        %  Assign points corresponding to postive values to 1
```

Create Plot in Matlab

$$i_{0\text{pulse}}(t) = \left[\left(0.8 - 0.8\exp(-1250t)\right)u(t) - \left(0.8 - 0.8\exp(-1250(t - 2\text{m}))\right)u(t - 2\text{m})\right]\text{amps}$$

>> tau = 1/1250; % Define time constant

>> t = [-0.1:(tau/50):(10*tau+2e-3)]; % Create time axis

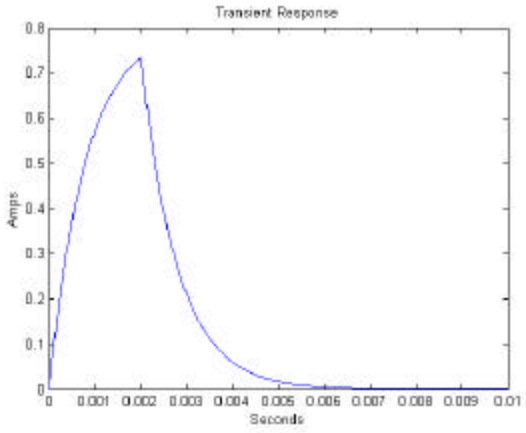
>> i0 = (.8-.8*exp(-t/tau)).*unit(t)-(.8-.8*exp(-(t-2e-3)/tau)).*unit(t-2e-3);

>> plot(t,i0)

>> xlabel('Seconds')

>> ylabel('Amps')

>> title('Transient Response')



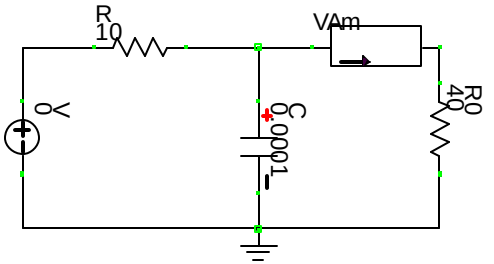
SPICE Simulation with Step Source

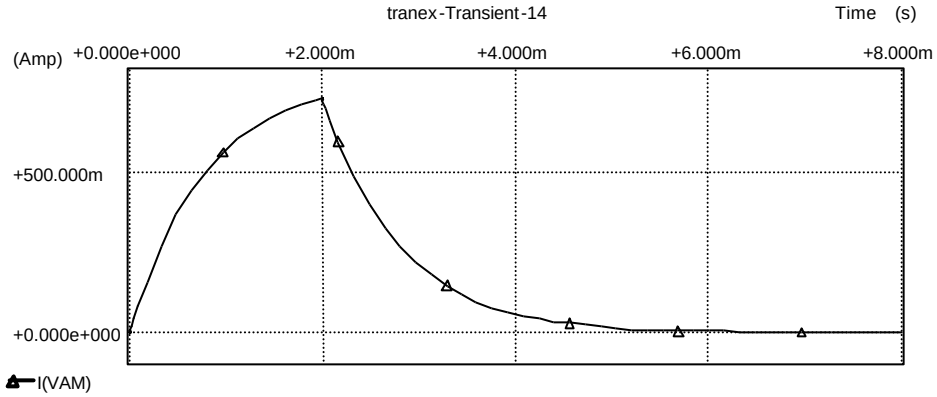
Do a SPICE Simulation to determine the pulse response for $i_o(t)$ the Previous circuit with input $v_s(t) = 40v(t)$ V, where pulse duration T is 2 ms:

- **To solve you must do a transient analysis, define the voltage source as piece-wise linear, and describe its transient properties.**

tranex-Transient-14-Table

TIME (s)	I(VAM) (Amp)
+0.000e+000	+0.000e+000
+10.000n	+99.999n
+10.840n	+109.104n
:	:
:	:
:	:
+7.285m	+972.534u
+7.445m	+795.710u
+7.605m	+651.035u
+7.765m	+532.665u
+7.925m	+435.817u
+8.000m	+396.630u





SPICE Simulation with Switch

Do a SPICE Simulation to determine the pulse response for $i_o(t)$ the circuit below with input $v_s(t)$ = 40 V, where pulse duration T is 2 ms:

- **To solve you must do a transient analysis, define auxiliary circuits for voltage sources and meters that control the opening and closing of the switches.**