# EE 422G - Signals and Systems Laboratory

## Lab 8 Band-Pass Modulation

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**Objectives:**
- Understand the models for signal modulation with real and complex sinusoids.
- Apply modulation to efficiently use available bandwidth for transmission.
- Implement an AM modulation system and examine impact of modulation index.

## 1. Background

Communication channels are restricted by their inherent physical properties, which are measured in terms of bandwidth and noise. Channel bandwidth is determined by properties of the physical medium or set by a protocol/regulation (such as is done by the FCC for most broadcast bands). Signals supported by frequencies almost entirely within the channel bandwidth can be transmitted reliably. Frequency content outside this range is attenuated. Therefore, in the process of sending signals from one point in physical space (source) to another (sink), the signals must be modified according to the available/allocated channel bandwidth. Modulation shifts baseband signal spectra to a different frequency range. In many cases modulation is implemented by multiplying a sinusoidal waveform with the baseband signals.

Modulation for real signal can be understood from the following trigonometric identity:

$$\sin(2\pi f t)\cos(2\pi f_c t) = \frac{1}{2}\left(\sin(2\pi(f+f_c)t) + \sin(2\pi(f-f_c)t)\right) \tag{1}$$

where $f_c$ is the modulating frequency and $f$ is the baseband signal frequency. Note the signal being modulated is shifted up and down the spectrum (double-sided) by the modulating frequency. Complex sinusoid modulation of another complex sinusoid, shifts the signal spectrum in only one direction:

$$\exp(j2\pi f t)\exp(j2\pi f_c t) = \exp(j2\pi(f_c+f)t) \tag{2}$$

Real-valued signals have double sided spectra (positive and negative frequencies) with even symmetry for the magnitude and odd symmetry for the phase. Because of this redundancy, only one side is needed to represent the unique signal information. A complex signal can have a one-sided spectrum if the real and imaginary parts of the time domain signal are related through the Hilbert transform.

A circuit implementation of a complex signal is usually accomplished by creating 2 channels where one is sifted 90 degrees with respect to the other. One channel is the real component and the other is the imaginary component. The advantage of using a complex representation is that the signal can be represented by a one-sided spectrum, effectively cutting its bandwidth in half. The one-sided spectrum concept is used in single-sideband modulation systems. While the complex signal system requires less bandwidth, it also requires more complexity for both the transmitter and receiver.

One of the simplest modulation schemes, Amplitude Modulation (AM), is used in commercial radio (AM band). The frequency ranges of radio stations are controlled by government regulations. For example a radio station can purchase a license to broadcast radio signals in the range of 720 kHz to 760 kHz, which limits the broadcast bandwidth to 40 kHz. Since the audio signal for radio ranges from about 20 Hz to 20 kHz, the radio transmitter *modulates* the baseband signal (limited to 20 kHz) to a higher frequency by effectively multiplying it with a sinusoid at 740 kHz. This results in a signal within the 720 to 760 kHz range, and contains the same information as the original baseband signal. A radio receiver *demodulates* the signal by multiplying it with a sinusoid to shift signal down in frequency and recover the original baseband signal through filtering out the higher frequency spectral images.

## 2. Pre-Laboratory Exercises

1. a) Based on the modulation property, sketch the spectrum of the modulated waveform:
$$s(t) = 500\,\text{sinc}(500t)\cos(2\pi 4000t)$$
   b) Sketch the spectrum of the demodulated waveform:
$$r(t) = s(t)\cos(2\pi 4000t)$$
   c) Describe the problems that would occur with the above modulation and demodulation process if the modulating frequency was only 400 Hz (rather than 4000 Hz).

2. a) Based on the complex modulation property, sketch the spectrum of the modulated waveform:
$$\hat{s}(t) = 500\,\text{sinc}(500t)\exp(-j2\pi 4000t)$$
   b) Sketch the spectrum of the demodulated waveform:
$$\hat{r}(t) = \hat{s}(t)\exp(j2\pi 4000t)$$
   c) Would this modulation/demodulation have similar problems as in the real modulation of Problem 1 if the modulating frequency was only 400 Hz (rather than 4000 Hz)?

## 3. Laboratory Exercises

1. For this exercise use the mfile function, named *simpmod.m* located at:
   http://www.engr.uky.edu/~donohue/ee422/mfiles/simpmod.m
   Modify the *simpmod.m* script to verify your sketches in the pre-lab Problems 1 and 2, which used the sinc function as the test signal. It would be best to have a symmetric time axis (i.e. positive and negative time values) so the sinc function appears symmetric about $t = 0$. Do not use a PSD (averaging over a hopping window), as in the original script, just use the FFT magnitude over a single segment of the waveform. **Show the plots to the TA and explain similarities and differences with the pre-lab sketches**.

2. For this exercise you will need to download an audio file sampled at 48kHz. There are 3 such files located at:

   http://www.engr.uky.edu/~donohue/ee422/Data/mfopt.wav
   http://www.engr.uky.edu/~donohue/ee422/Data/twNspec.wav
   http://www.engr.uky.edu/~donohue/ee422/Data/impNtrans.wav

   You just need to use one of these for this exercise. Since modulation shifts the waveform on the frequency axis, these waveforms will be upsampled to 96kHz to raise the Nyquist (aliasing) frequency to 48kHz. So any signal frequencies shifted above 48kHz will fold back in the spectrum causing distortion. Make sure to avoid this when modulating the signals. The *resample* command in Matlab can be used for both upsampling and downsampling. You need to ensure the Nyquist frequency is high enough to prevent aliasing of the significant signal energy after modulation.

   You will need a hidden mfile function, referred to as a pfile, located at:
   http://www.engr.uky.edu/~donohue/ee422/mfiles/bbchan.p

   The file *bbchan.p* is an executable Matlab file that operates as an mfile function, except the contents cannot be viewed or edited. It was written to simulate transmission over a band-pass channel for which the pass-band parameters are unknown. The channel also adds 10 mW of white Gaussian noise so the received signal will sound noisy (note the weaker the input signal is, relative to 10mW, the noisier the received signal will sound). The wave files can be loaded into the Matlab workspace with:
   >> [y, fs] = audioread('filename');
   The audio file samples are included in vector *y* and sampling rate is in scalar *fs*. The audio file can be played with the command:
   >> soundsc(y,fs)

   For this exercise upsample the waveform to 96kHz as was done in the *simpmod.m* script to limit aliasing during the modulation and demodulation process. Use this sampling rate for the following problems:

   (a) determined the passband for the channel

     (b) modulate the audio signal *y* (i.e. assign it to some new variable *ym*) so it will efficiently pass through the channel with the least distortion

     (c) send the modulated signal over the channel (i.e. *r = bbchan(ym,fs)* and demodulate.

     (d) Filter the received signal, *r*, to reduce noise.

     (e) Compare the spectral magnitude (can use the PSD with averaging for this) of the original to the modulated, transmitted (output of channel), and demodulated signal.

**Use these plots to present to the TA an** explanation of the process used to find the pass-band range for the channel, along with a description of the modulation scheme and the filter used to reduce noise. **Demonstrate to the TA that your modulation process worked** by playing back the sound file before and after passing through the channel.

3. An AM waveform can be represented mathematically by:

$$s(t) = A_c[1 + \delta\, m(t)]\cos(2\pi f_c t) \qquad (4)$$

where $m(t)$ is baseband or message signal, and $A_c \cos(2\pi f_c t)$ is the carrier. The δ parameter controls the percent modulation (related to the modulation index), which allows for a percentage of the carrier to be present in the modulation signal. The reason for modulating this way is to allow for inexpensive (simple) demodulators. For a radio broadcast over a region, few transmitters are needed; however, many radio receivers are needed for the population listening to the radio programming. So the most cost effective approach is to make the receivers as inexpensive as possible. If $1 > |\delta\, m(t)|$ for all *t*, then a simple demodulation scheme can be used. The message signal will effectively be the envelope on the carrier. Therefore, rectify the received waveform and low-pass filter it to recover the envelope. This is given by:

$$r(t) = h(t) * |s(t)| \qquad (5)$$

where * denotes convolution and $h(t)$ is a low-pass filter with a cutoff frequency high enough to include the baseband signal without significant distortion. This demodulator can be implemented with a simple circuit using a coil, a diode, and a capacitor. For this exercise, amplitude-modulate one of the audio signals with a 22 kHz modulating. I recommend writing a function that inputs a message signal, carrier frequency, and modulation index and outputs the modulated waveform. This way you can vary the carrier frequency and modulation index to observe the results. Then demodulate it by applying a low-pass filter to the rectified (absolute) values of modulated signal. I recommend writing function for this as well that inputs the modulated waveform, and low-pass cut off. The output is then the demodulated signal. Modulate and demodulate the signal using a modulation index of 0.1 and listen to the demodulate waveform. Do the same for a modulation index of 10. **Play these for the TA.** Optional: To help with understanding try (or debugging) modulating a simple sinusoid carrier at 2000Hz and look at resulting spectra.