Statics of Structural Supports

TYPES OF FORCES

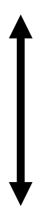
External Forces ≡ **actions of other bodies on the structure under consideration.**

Internal Forces ≡ **forces and couples exerted on a member or portion of the structure by the rest of the structure. Internal forces always occur in equal but opposite pairs.**

Supports

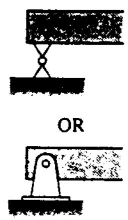
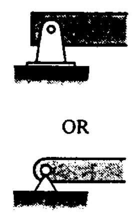
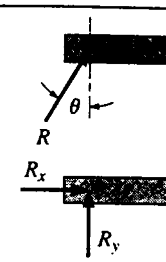
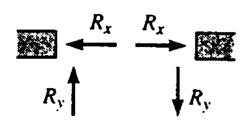
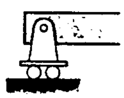
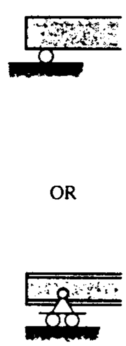
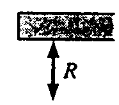
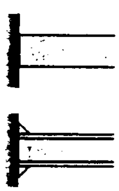
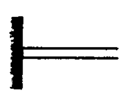
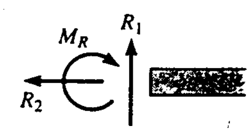
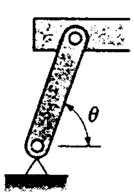
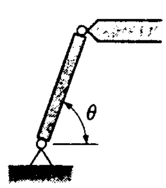
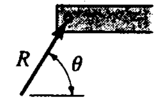
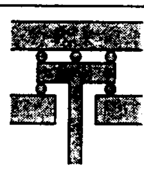
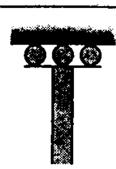
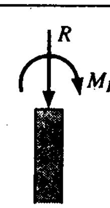
Different types of structural supports are shown in Table 1. Some physical details for the idealized support conditions of Table 1 are shown in Figs. 1 – 5.

NOTE:



Structural roller supports are assumed to be capable of resisting normal displacement in either direction

Table 1. Idealized Structural Supports

Type	Sketch	Symbol	Movements Allowed or Prevented	Reaction Forces	Unknowns Created
(*a*) Pin			*Prevented:* horizontal translation, vertical translation *Allowed:* rotation	A single linear force of unknown direction or equivalently A horizontal force and a vertical force which are the components of the single force of unknown direction	
(*b*) Hinge			*Prevented:* relative displacement of member ends *Allowed:* both rotation and horizontal and vertical displacement	Equal and oppositely directed horizontal and vertical forces	
(*c*) Roller			*Prevented:* vertical translation *Allowed:* horizontal translation, rotation	A single linear force (either upward or downward*)	
(*d*) Rocker					
(*e*) Elastomeric pad					
(*f*) Fixed end			*Prevented:* horizontal translation, vertical translation, rotation *Allowed:* none	Horizontal and vertical components of a linear resultant; moment	
(*g*) Link			*Prevented:* translation in the direction of link *Allowed:* translation perpendicular to link, rotation	A single linear force in the direction of the link	
(*h*) Guide			*Prevented:* vertical translation, rotation *Allowed:* horizontal translation	A single vertical linear force; moment	

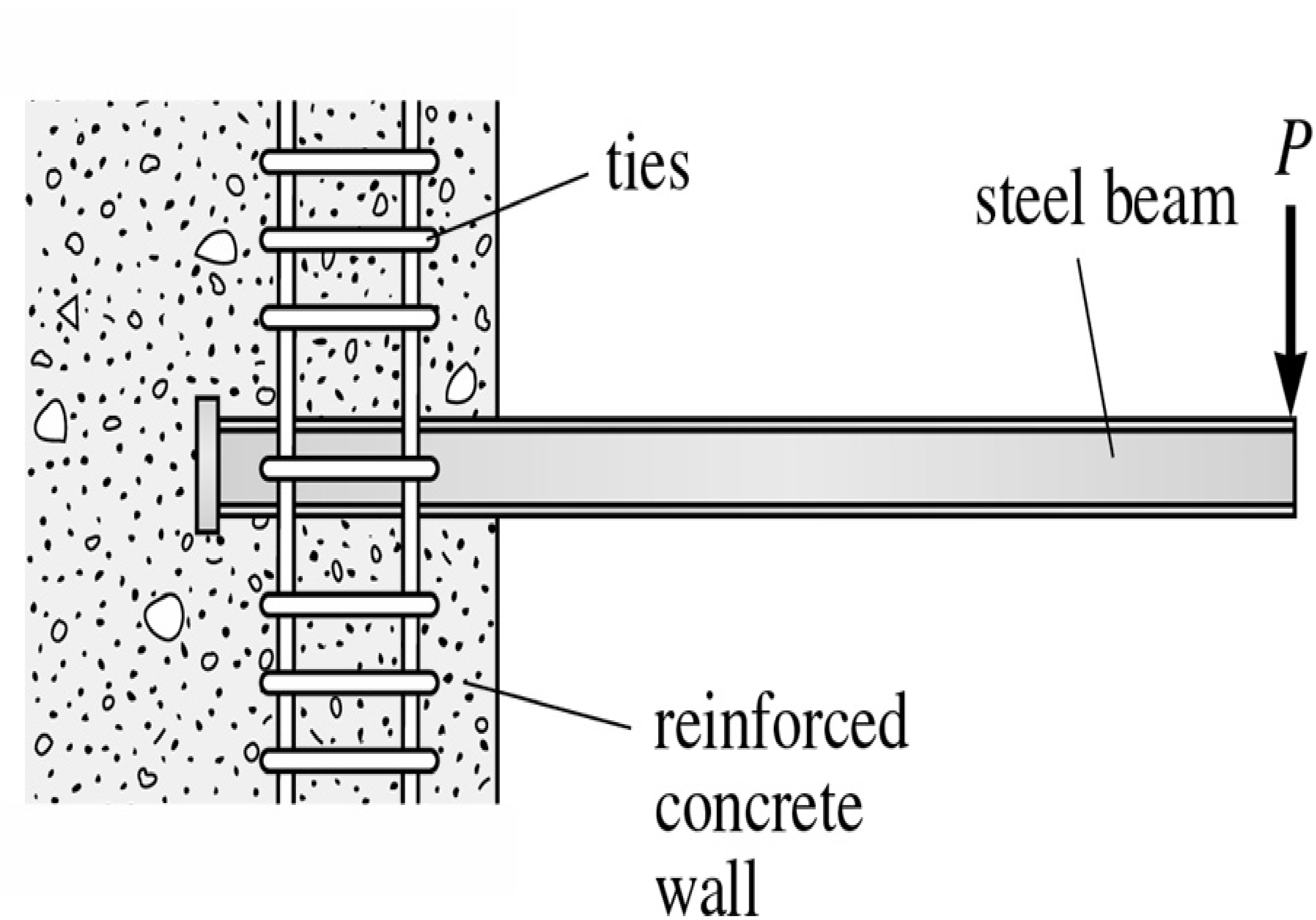


Figure 1. Example Fixed Steel Beam Support

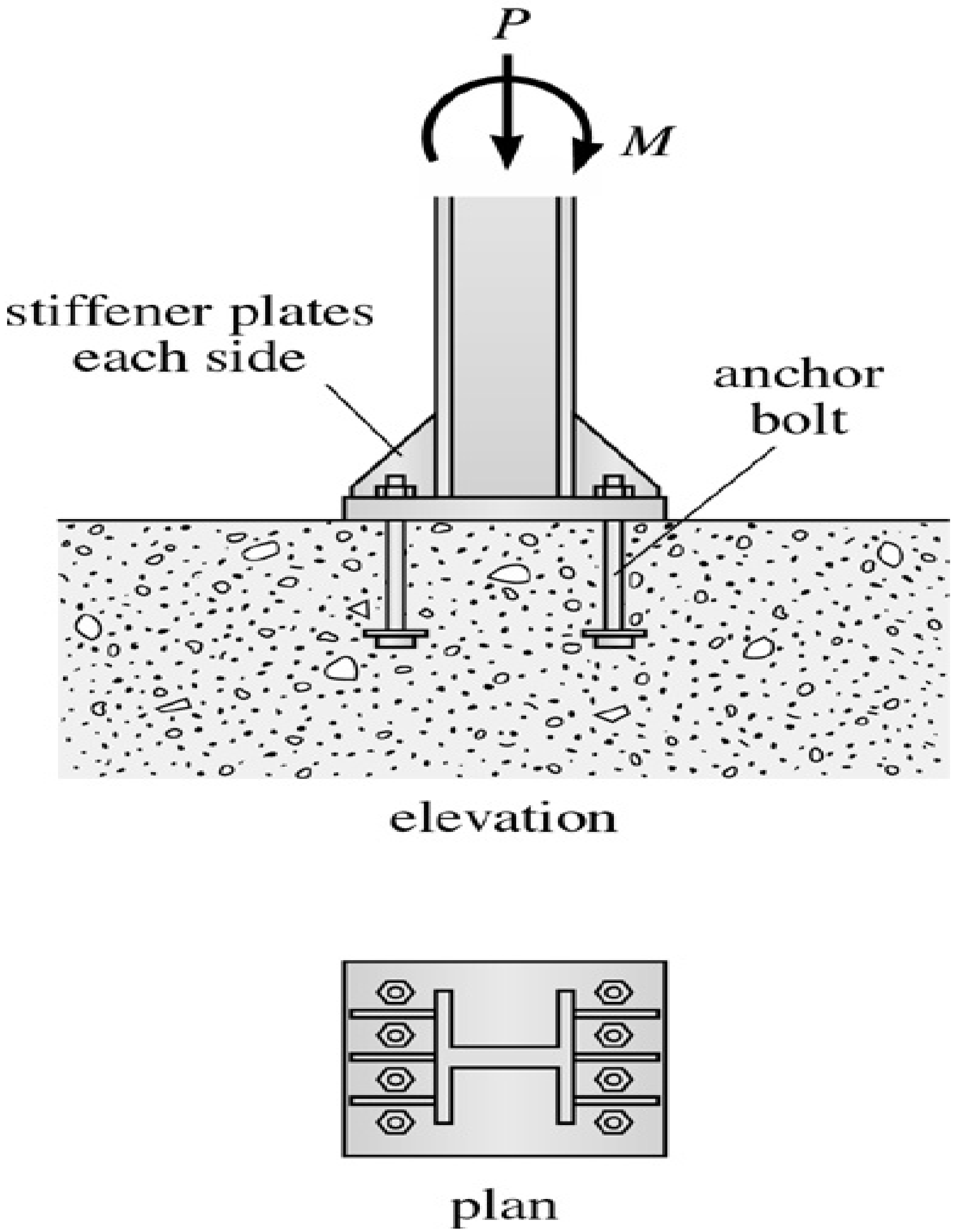


Figure 2. Example Fixed Steel Column Support

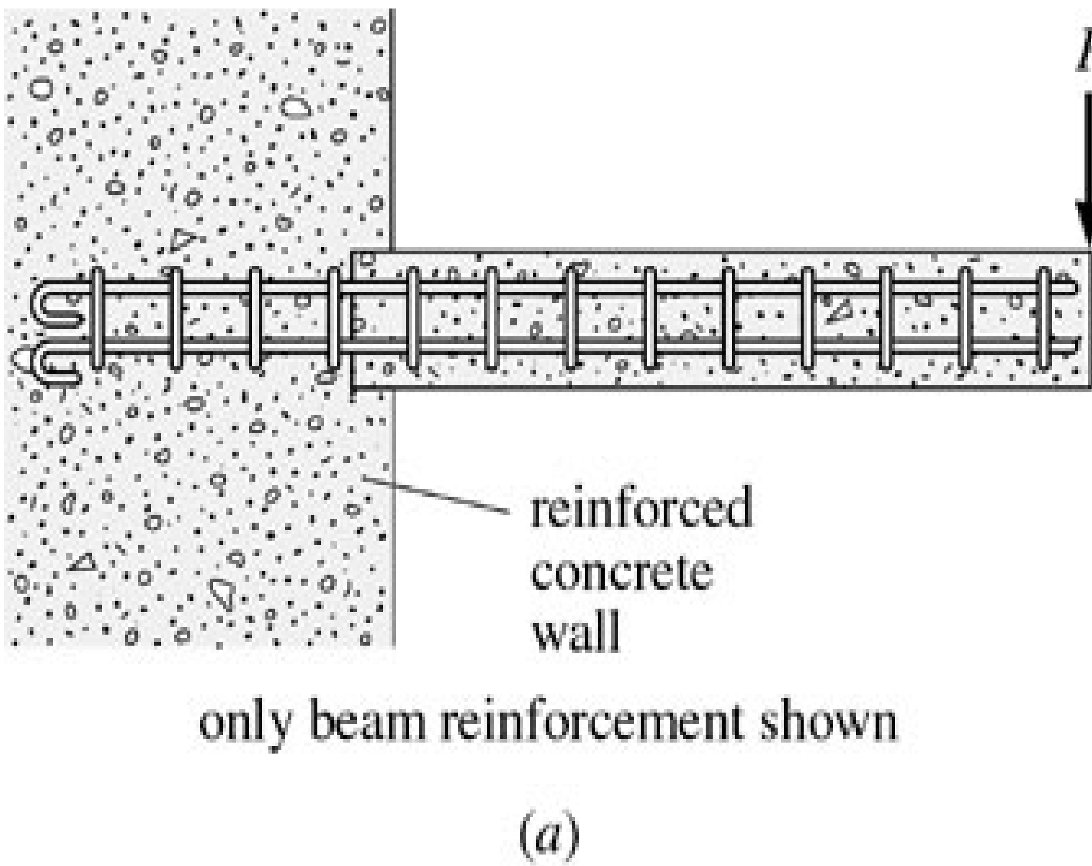


Figure 3. Example Fixed Concrete Beam Support

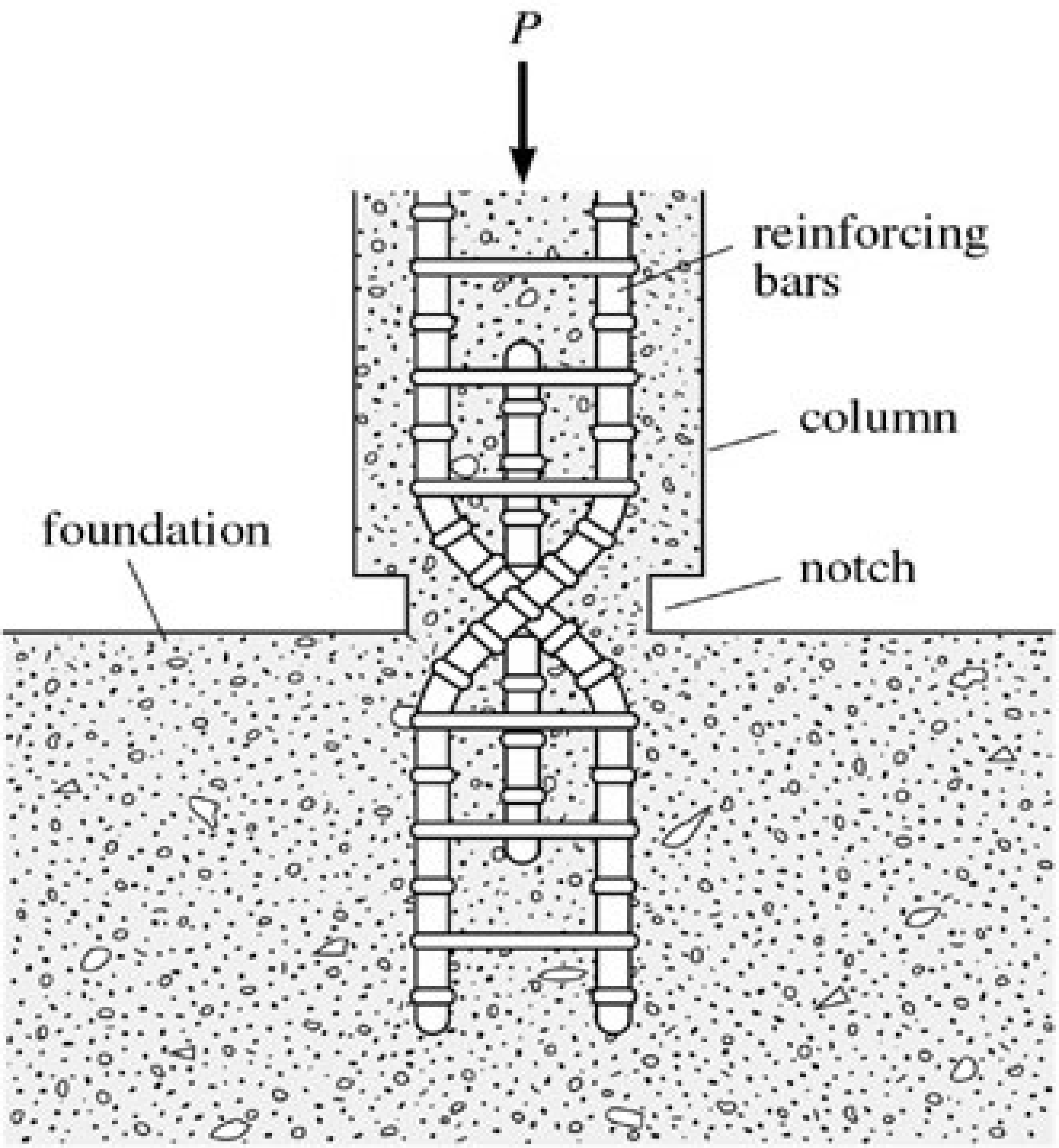


Figure 4. Example Simply Supported Concrete Column Support

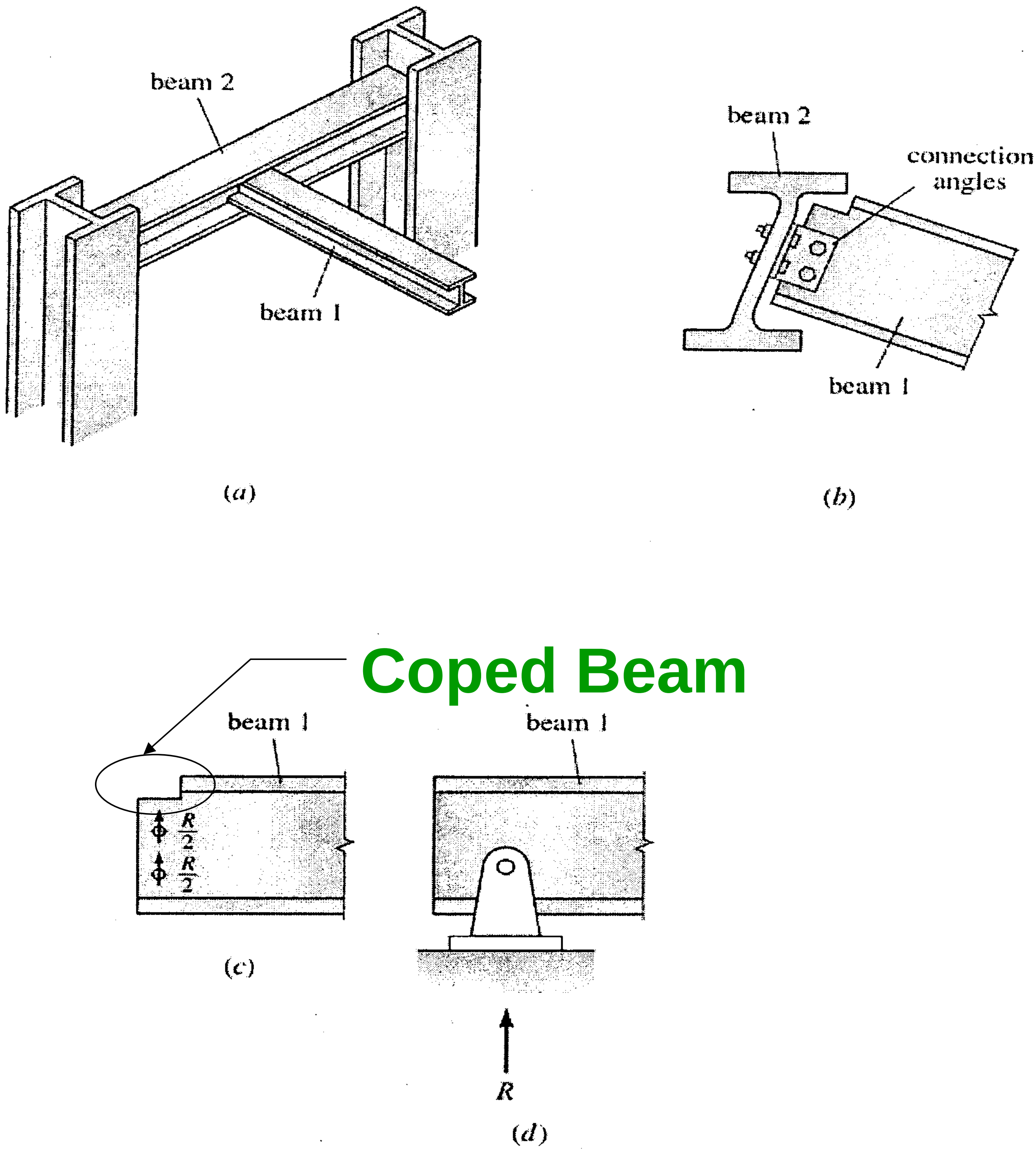


Figure 5. Example Simply Supported Floor Beam (beam 1) to Girder (beam 2) Conditions

Equations of Static Equilibrium

A structure is considered to be in equilibrium if, initially at rest, it remains at rest when subjected to a system of forces and couples. If a structure is in equilibrium, then all its members and parts are also in equilibrium.

For a plane structure lying in the xy plane and subjected to a coplanar system of forces and couples, the necessary and sufficient conditions for equilibrium are:

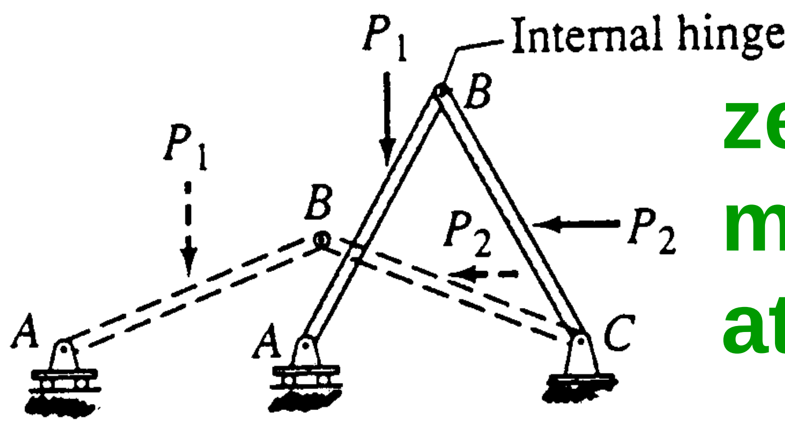
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_z = 0$$

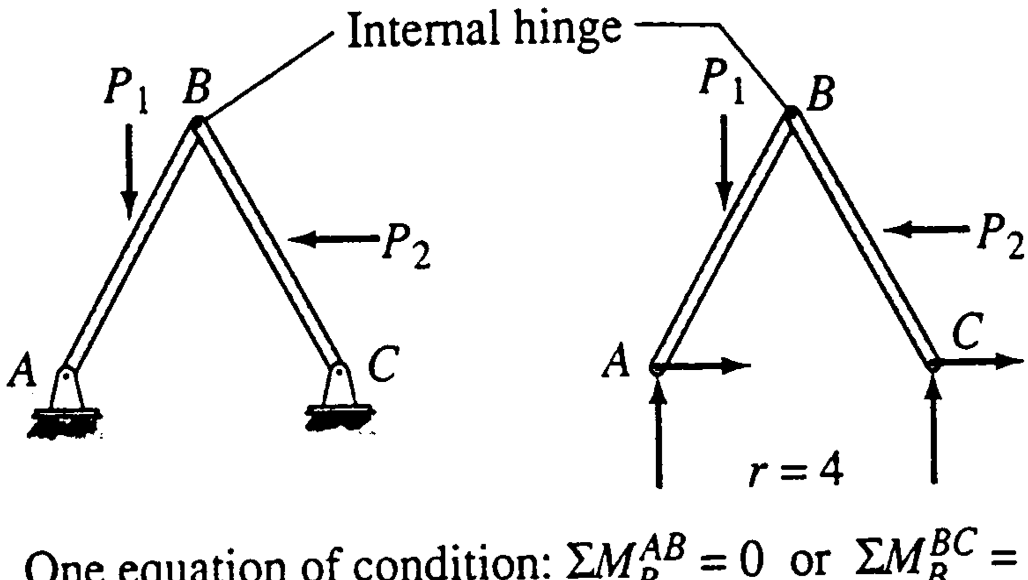
These three equations are referred to as the static equations of equilibrium of plane structures.

Equations of condition involve known equilibrium results due to construction.



zero moment at hinge

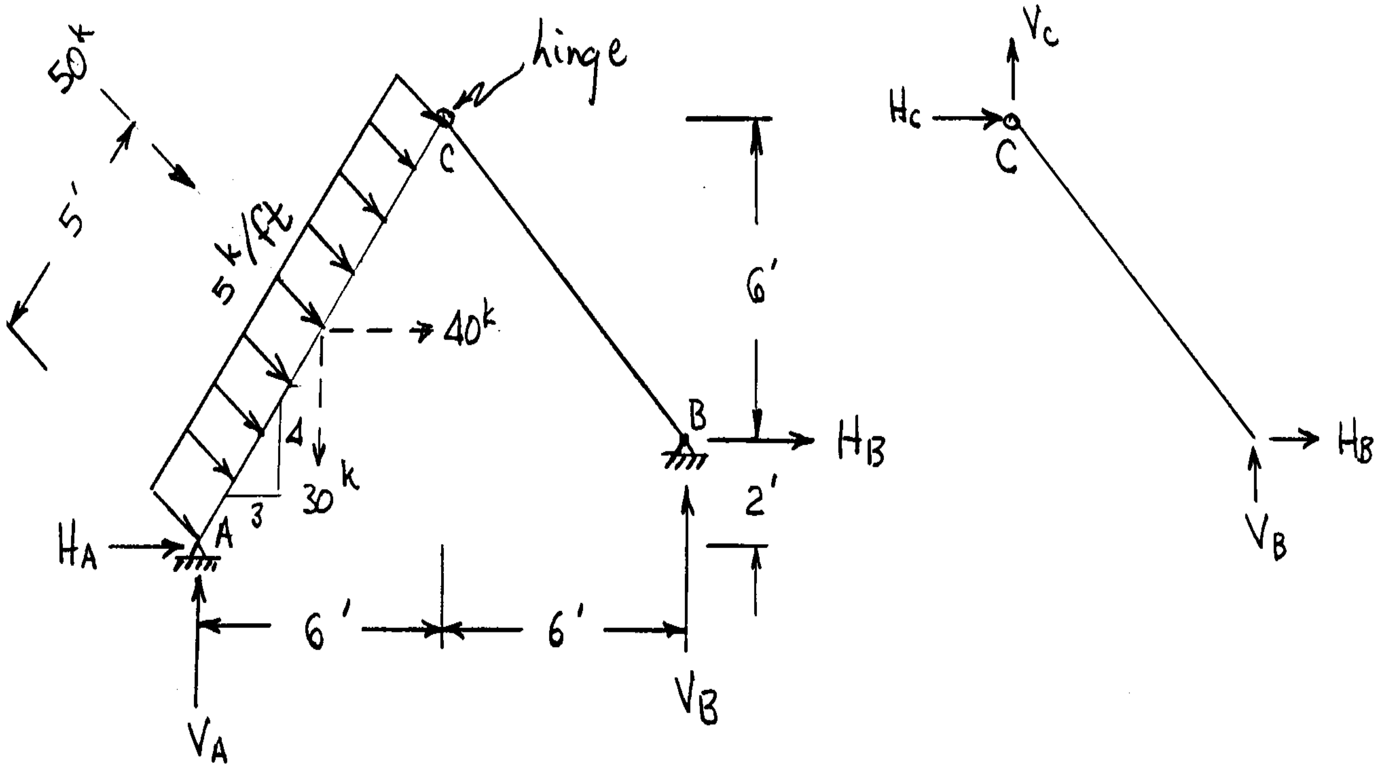
(a)



One equation of condition: $\Sigma M_B^{AB} = 0$ or $\Sigma M_B^{BC} = 0$

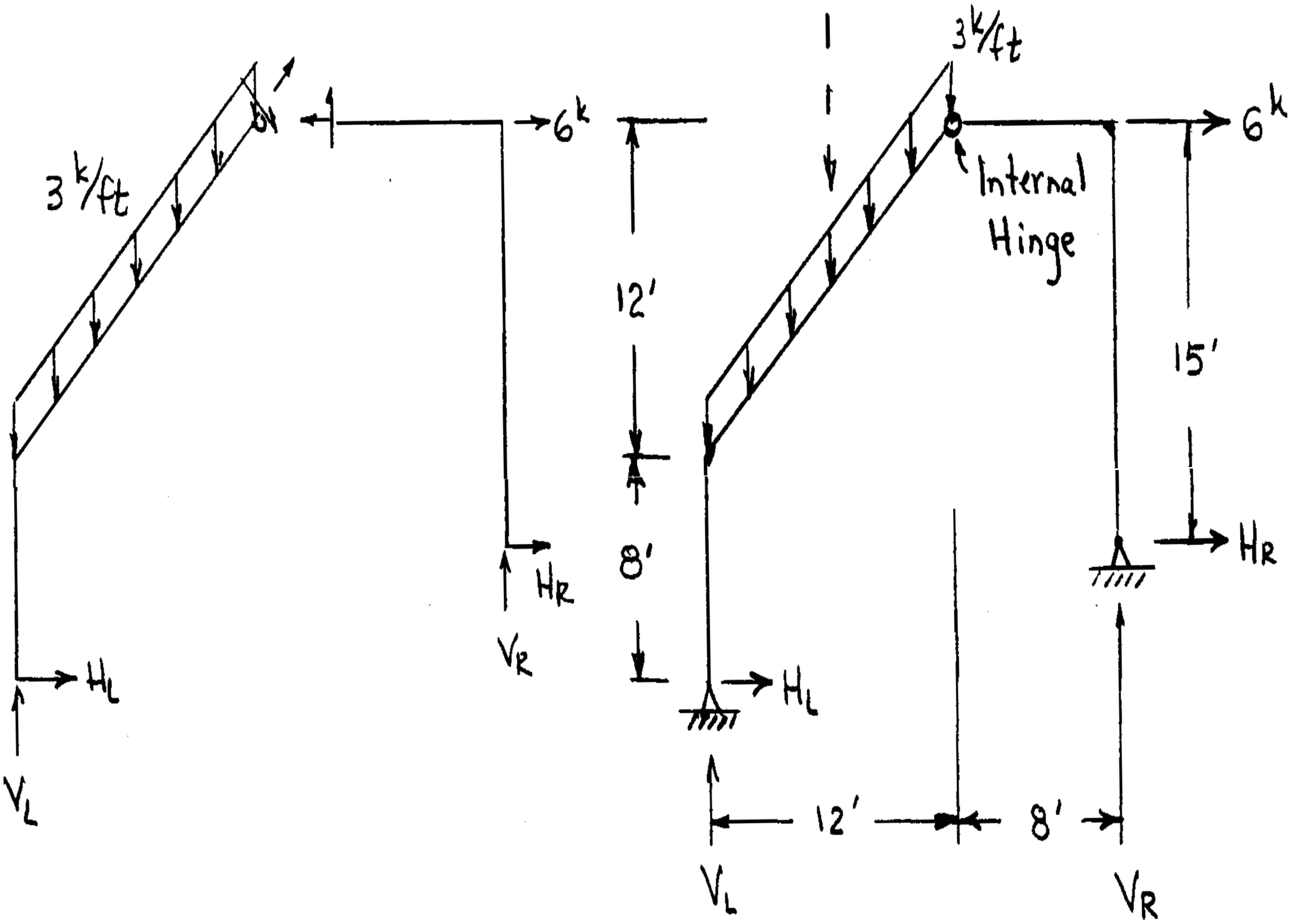
(b)

Example – Calculate the Support Reactions



Example – Calculate the Support Reactions

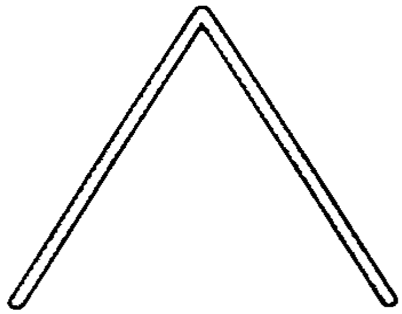
R_{UL} = 50.91 kips



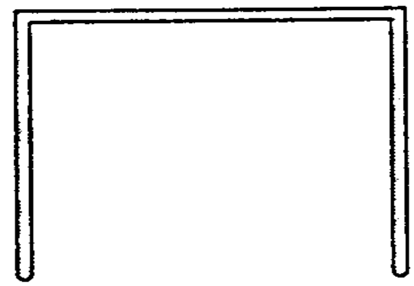
Influence of Reactions on Stability and Determinacy of Structures

Internally Stable (rigid) ≡ structure maintains its shape and remains a rigid body when detached from the supports.

Internally Unstable ≡ structure cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally.



(a)



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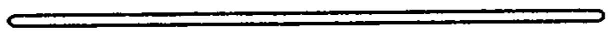
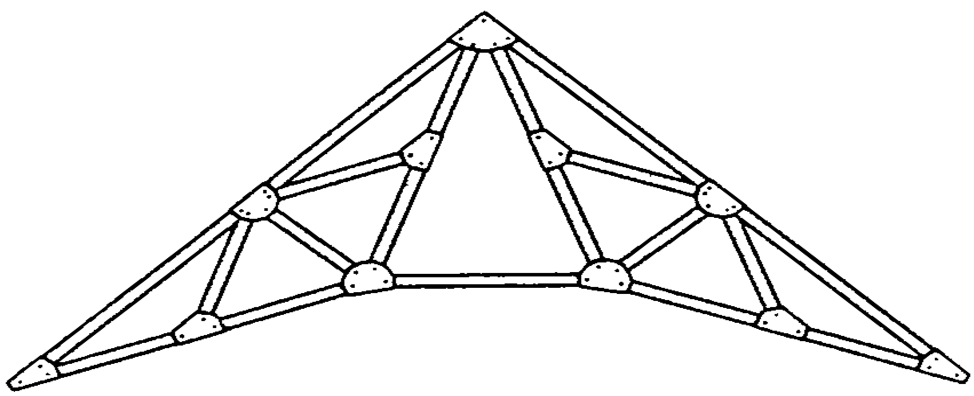
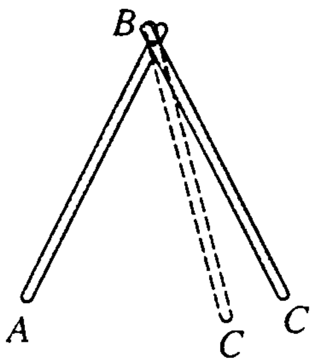


(c)

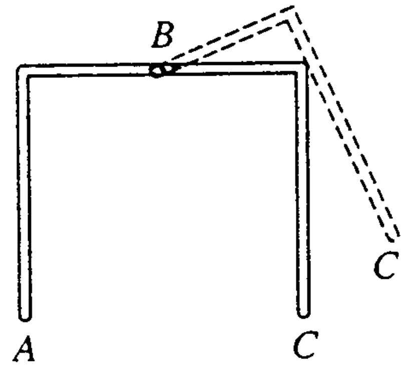


(d)

Examples of Internally Stable Structures



(a)



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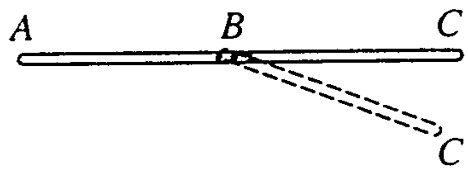

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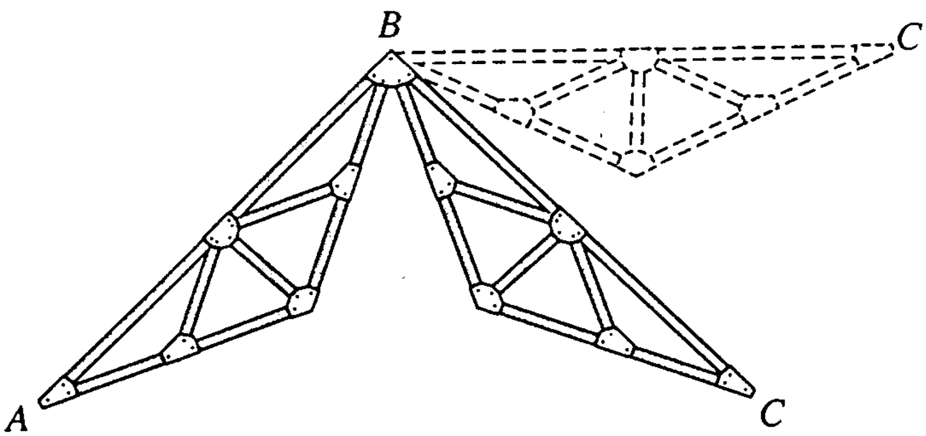


(d)

Examples of Internally Unstable Structures

Statically Determinate Externally ≡ If the structure is internally stable and if all its support reactions can be determined by solving equations of equilibrium.

Statically Indeterminate Externally ≡ If the structure is stable and the number of support reactions exceeds the number of available equilibrium equations.

External Redundants ≡ number of reactions in excess of those necessary for equilibrium, referred to as the *degree of external indeterminacy*.

Summary – Single Rigid Structure:

$R < 3$ Structure is statically unstable externally

$R = 3$ Structure may be statically determinate externally

$R > 3$ Structure is statically indeterminate externally, but may not be stable

R $\equiv$ number of support reactions

Summary – Several Interconnected Rigid Structures:

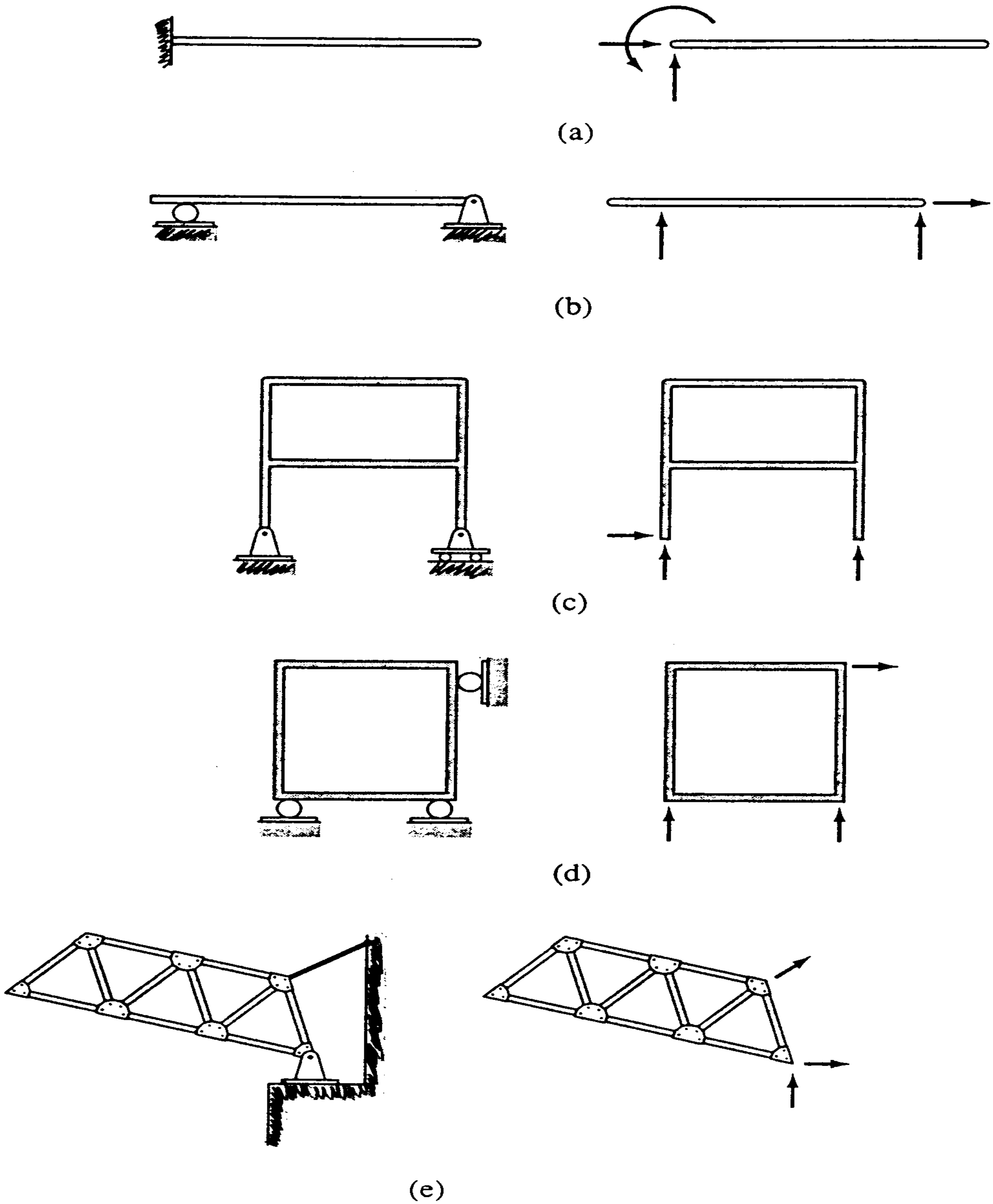
$R < 3+C$ Structure is statically unstable externally

$R = 3+C$ Structure may be statically determinate externally

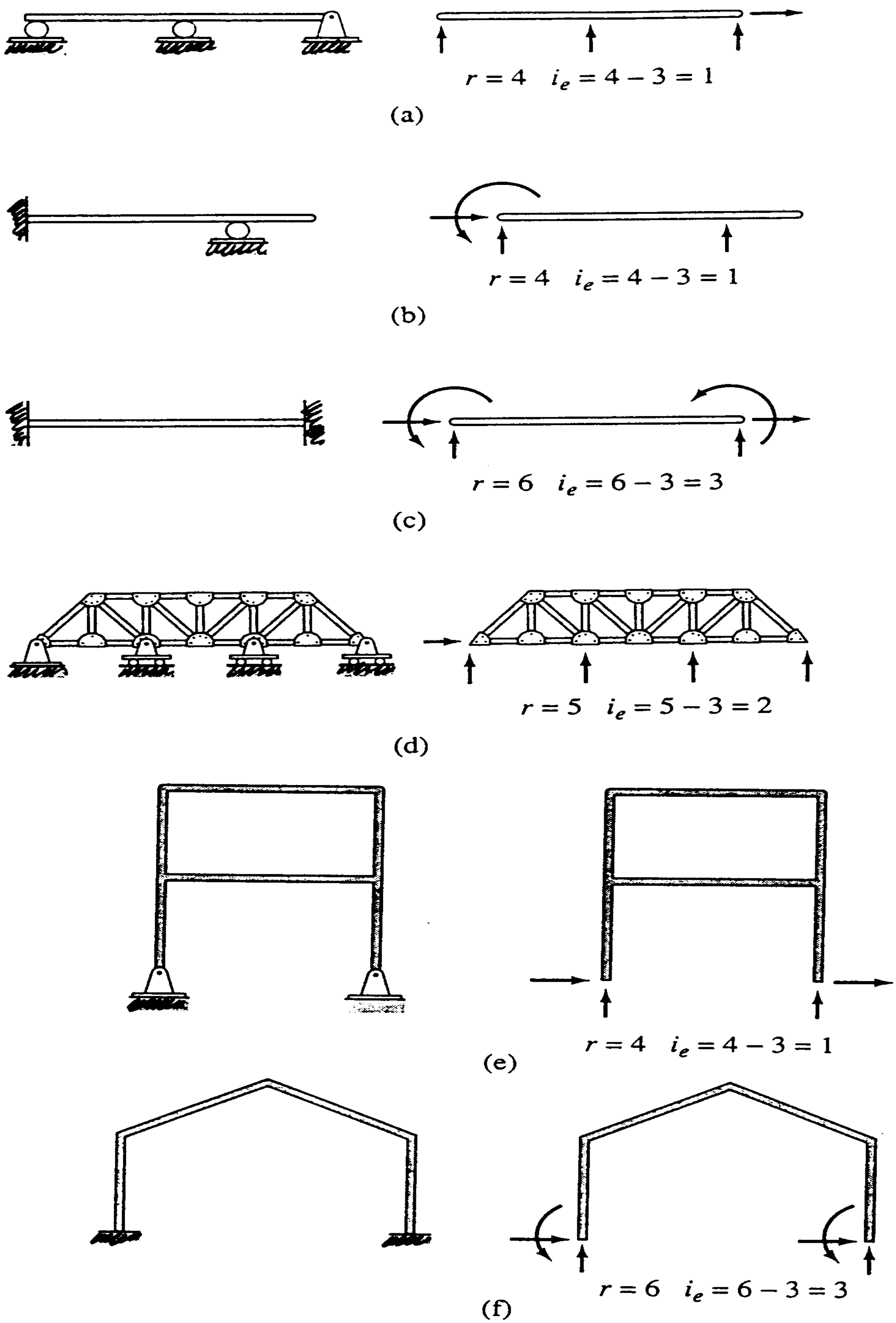
$R > 3+C$ Structure is statically indeterminate externally, but may not be stable

$C \equiv$ number equations of conditions

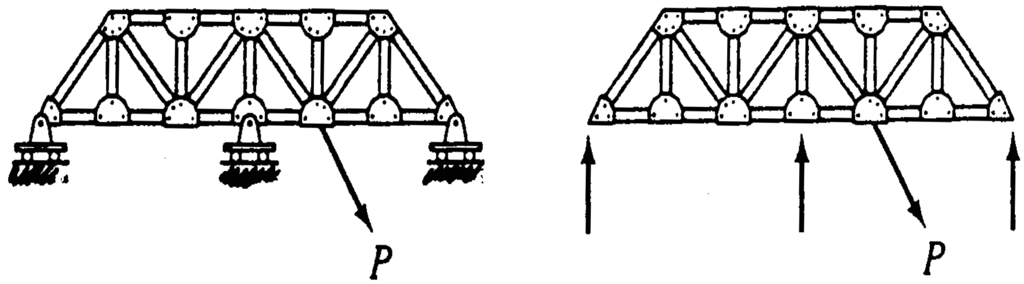
$I_e = R - (3 + C)$
$\equiv$ degree of external indeterminacy



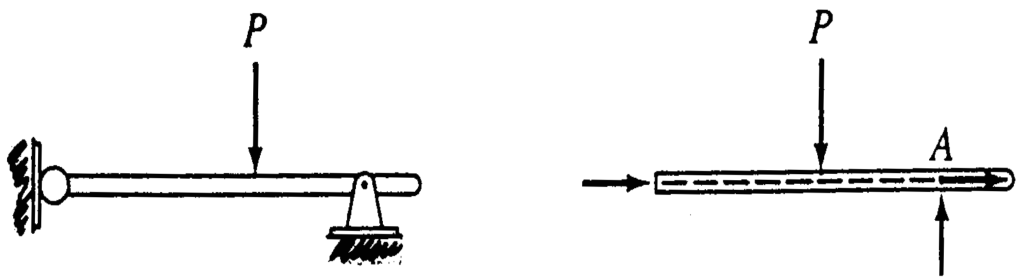
Examples of Externally Statically Determinate Plane Structures



Examples of Statically Indeterminate Plane Structures

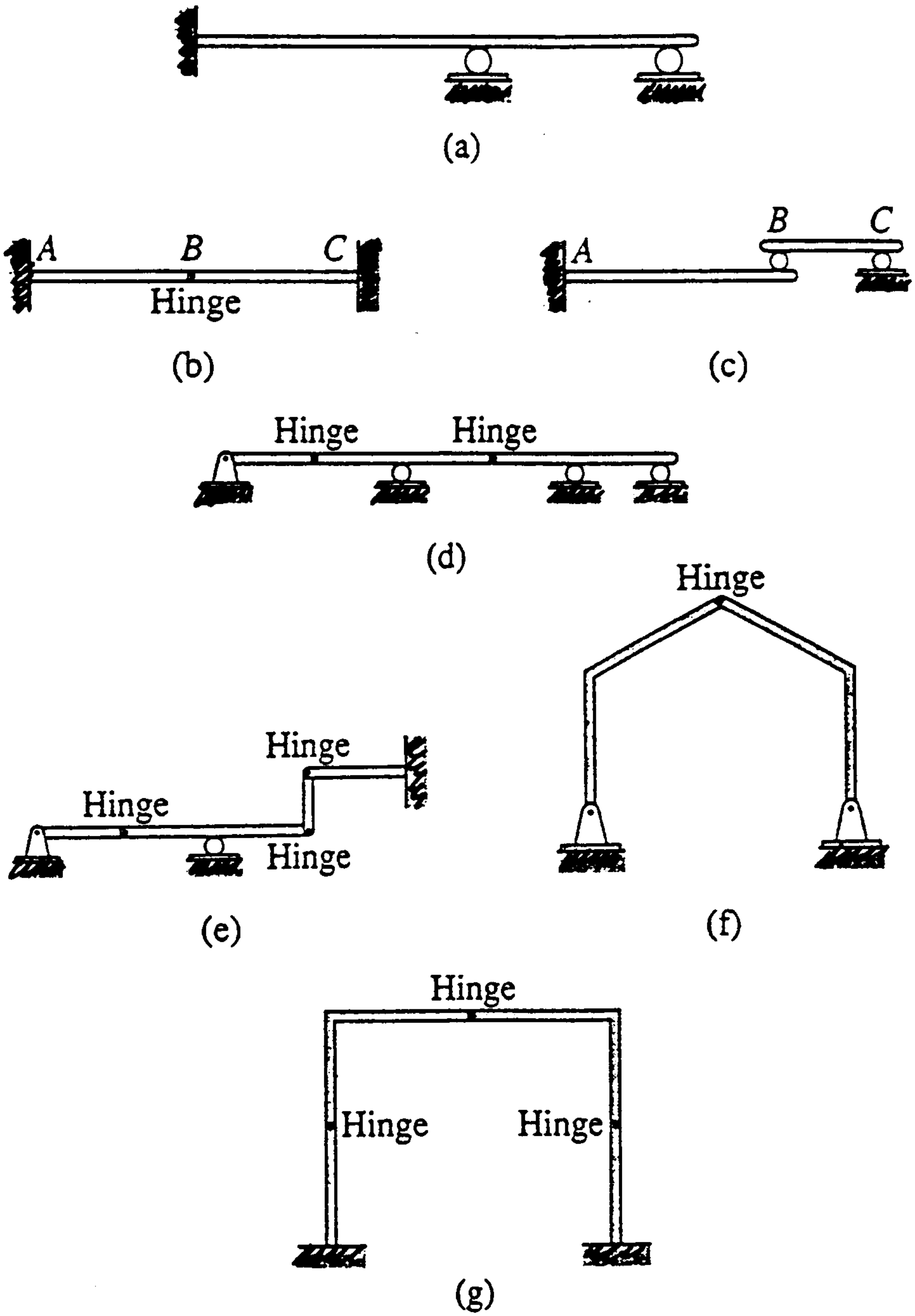


(a)



(b)

Reaction Arrangements Causing External Geometric Instability in Plane Structures



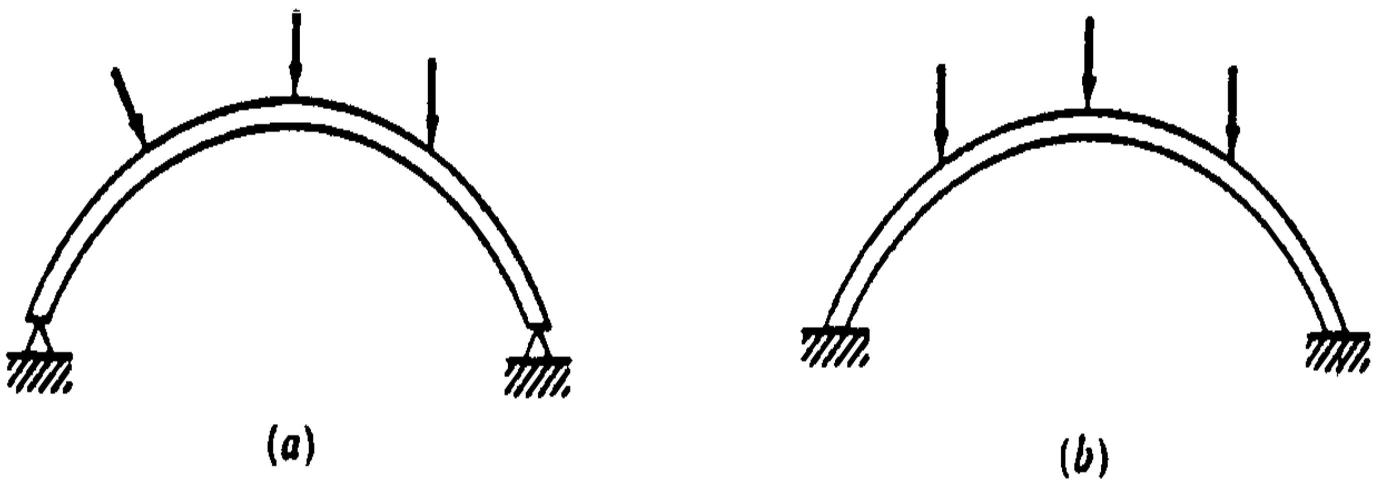
Example Plane Structures with Equations of Condition

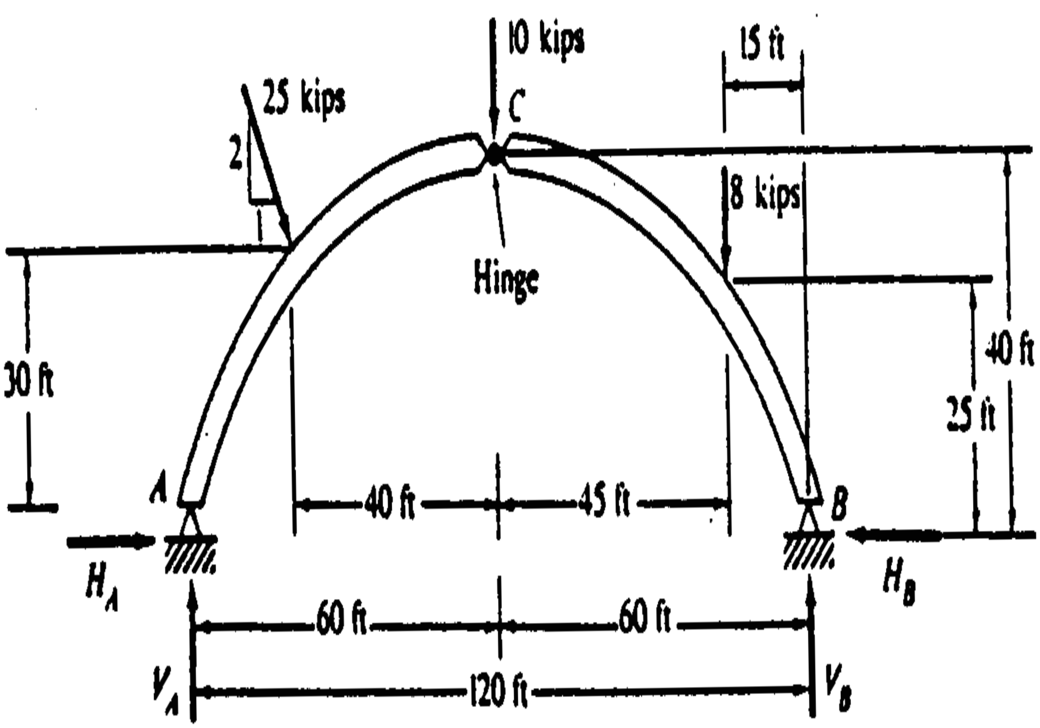
INTERIOR HINGES IN CONSTRUCTION

Interior hinges (pins) are often used to join flexural members at points other than support points, e.g., connect two halves of an arch structure and in cantilever bridge construction. Such structures are more easily manufactured, transported, and erected. Furthermore, interior hinges properly placed can result in reduced bending moments in flexural systems, and such connections may result in a statically determinate structure.

Arch Structures

Arch structures are usually formed to support gravity loads which tend to flatten the arch shape and thrust the supported ends outward. Hinge or fixed-end supports are generally used to provide the necessary horizontal displacement restraint. The horizontal thrust forces at the supports acting with the vertical loading tend to develop counteracting moments that result in low bending stresses.

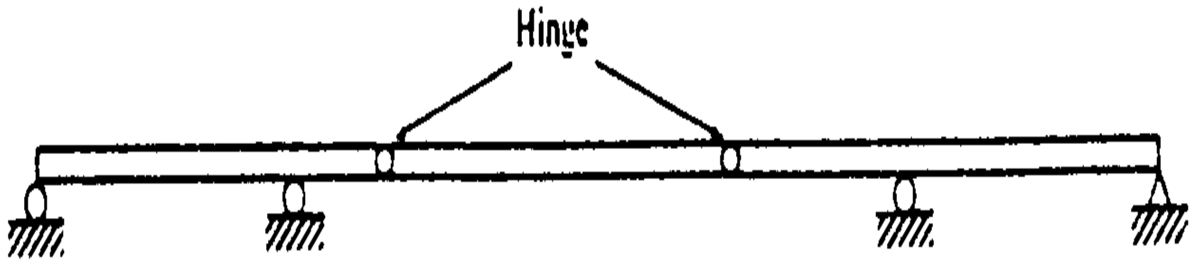




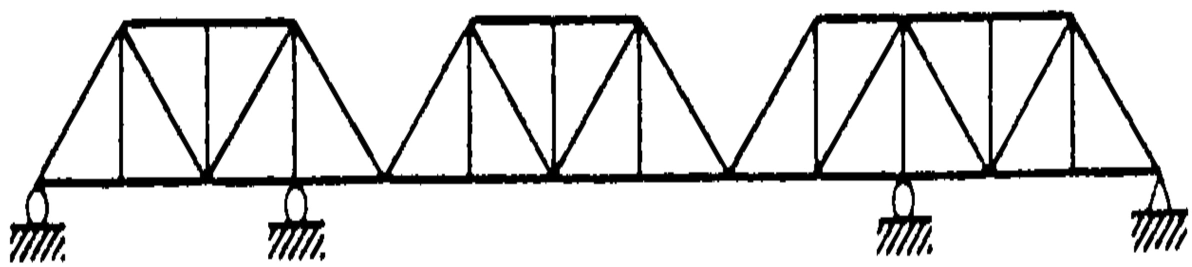
Arch Structure with Interior Hinge

Cantilever Construction

Cantilever construction represents a design concept that can be used for long span structures. If spans are properly proportioned, cantilever construction can result in smaller values of the bending moments, deflections, and stresses as compared with simple support construction.



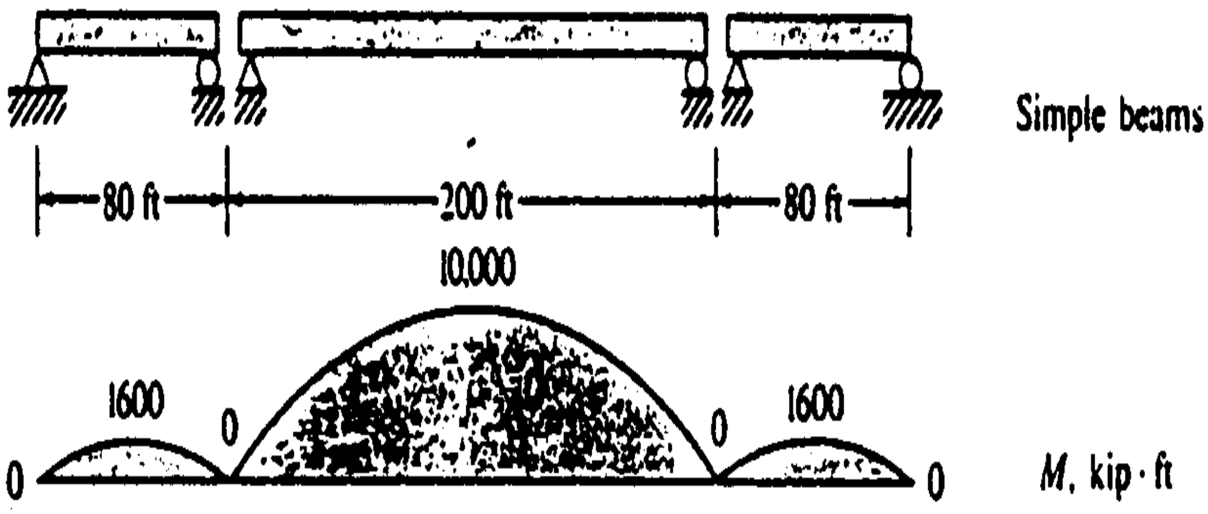
(*a*) Cantilever construction



(*b*) Cantilever truss construction

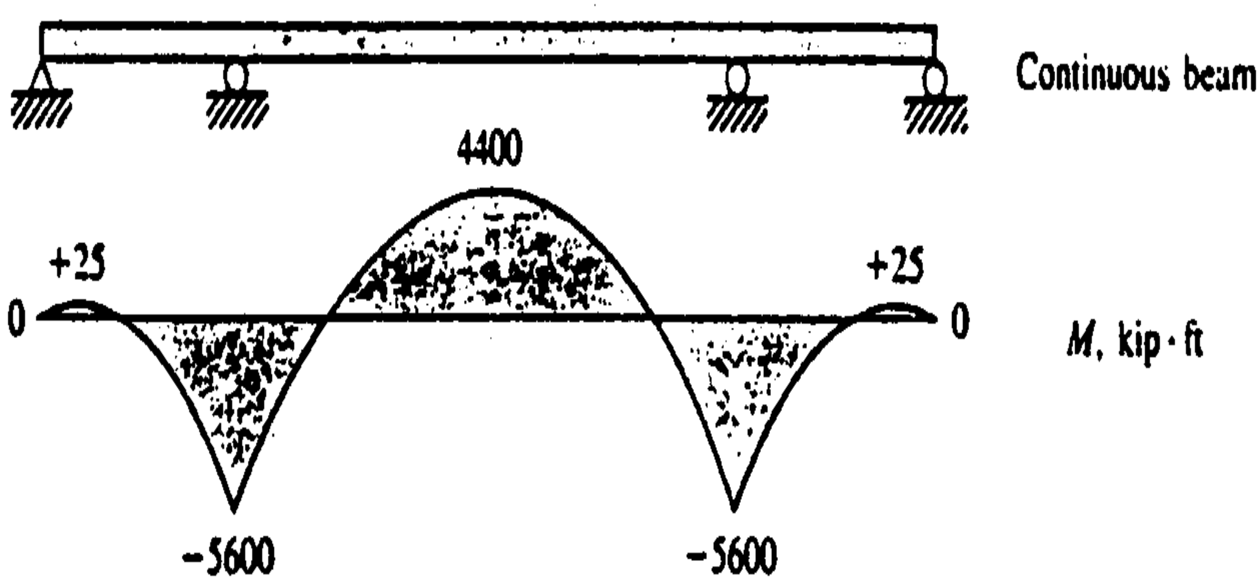
Examples of Cantilever Construction

The following figures show a typical highway overpass structure designed as a series of simple spans (a), a statically indeterminate continuous beam (b), and a canti-levered construction beam (c) along with their respective bending moment diagrams for a uniform load of 2 kips/ft. Note that the bending moments are most evenly divided into positive and negative regions for the three-span contin-uous beam and that the location of the internal hinges for the canti-levered constructed bridge resulted in a more even moment distribution as compared to the overpass analyzed as three simple spans.



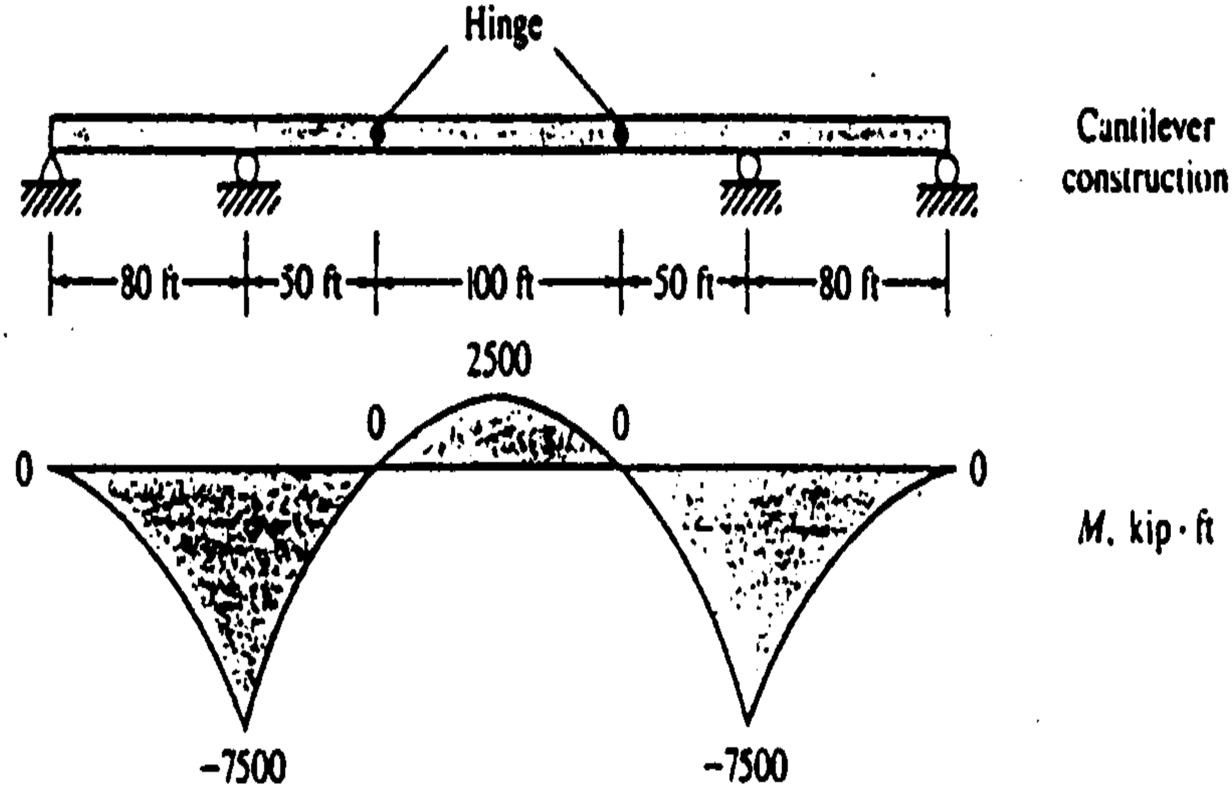
(a)

Simply Supported Spans



(b)

Continuous Spans



(c)

Cantilever Construction

Movement of the two internal hinges towards the interior supports results in a reduction of the negative moment magnitudes at the supports and an increase in the mid-span positive bending moment. Ideal placement occurs when the each interior hinge is approximately 109 ft from an end support, this location of the internal hinges results in a maximum negative and positive bending moments of 5000 ft-kips.

Cables

Use to support bridge and roof structures; guys for derricks, radio and transmission towers; etc. Assumed to only resist loads that cause tension in the cables.

Shape of cables in resisting loads is called **funicular**.

Resultant cable force is

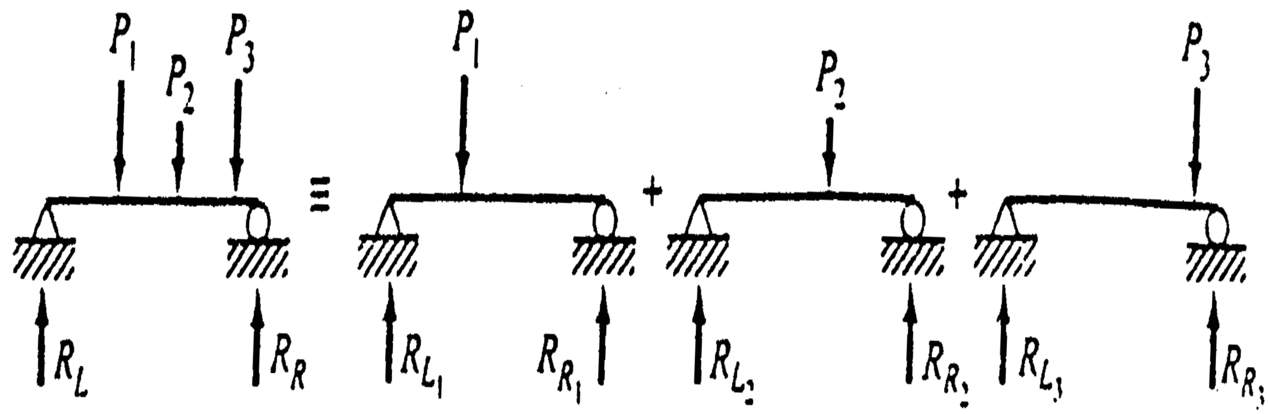
$$T = \sqrt{H^2 + V^2}$$

where **H** = horizontal cable force component and **V** = vertical cable force component.

Principle of Superposition ≡ on a linear elastic structure, the combined effect of several loads acting simultaneously is equal to the algebraic sum of the effects of each load acting individually.

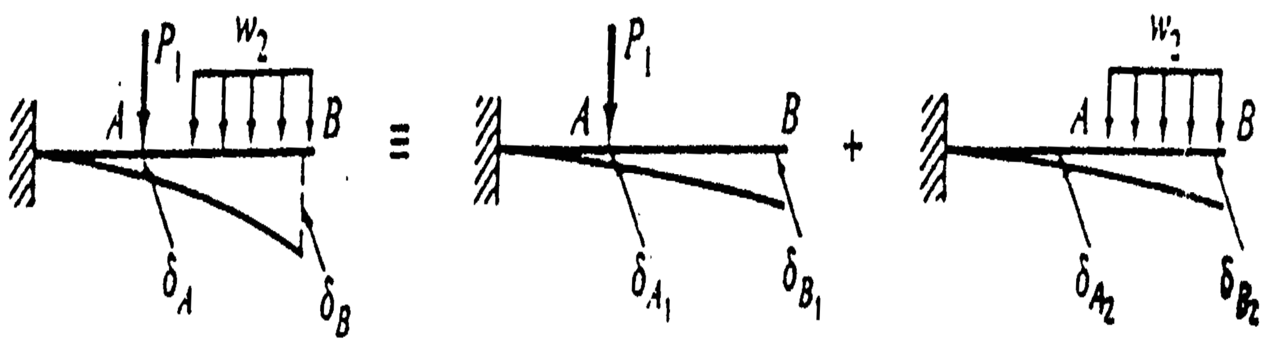
Principle is valid for structures that satisfy the following two conditions:
(1) the deformation of the structure must be so small that the equations of equilibrium can be based on the undeformed geometry of the structure; and
(2) the structure must be composed of linearly elastic material.

Structures that satisfy these two conditions are referred to as **linear elastic structures**.



$$R_L = R_{L_1} + R_{L_2} + R_{L_3}$$

$$R_R = R_{R_1} + R_{R_2} + R_{R_3}$$



$$\delta_A = \delta_{A_1} + \delta_{A_2}$$

$$\delta_B = \delta_{B_1} + \delta_{B_2}$$