Depolarization of radiation by non-absorbing foams

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Abstract

A Monte Carlo Ray-Tracing technique is developed to investigate the depolarization of radiation by foams simulated as layers of air-bubble-laden substrates. Angular and radial profiles of reflection and transmittance are predicted for one-dimensional media subjected to a collimated, polarized light beam. The effects of different bubble sizes, separation distance distributions between bubbles, and medium thickness are considered. Fresnel reflections at the boundaries of bubbles are accounted for using a ray-tracing approach. Calculations are performed to determine vertical and horizontal polarization components of both radial and angular profiles of reflection and transmission. It is shown that if the polarized reflection and transmission data can be obtained from carefully conducted experiments, they can be effectively used to diagnose the changes in the structure of foams. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Foams forming on liquid suspensions can be beautiful, with ever changing shapes and colors. We observe them on fresh brewed coffee, frothed milk, beer, molten butter and molten glass. Foams and cellular materials bear practical importance in many applications. Food industry always deals with foams and cellular structures, from chocolates to cereals to rising bread. The foamy layer on molten glass may act as an insulator that significantly alters the energy transfer mechanism from combustion gases to the load. In biomedical applications, the laser energy transfer from miniaturized probes to tissue may be affected significantly due to the formation of foams on the tissue.

Gibson and Ashby [1] discuss the structures and physical properties of a wide range of cellular solids and foams. According to them, the cell size of foams can vary from 10 nm for single-crystal
anodized silicon to 10 mm for paper–phenolic resin honeycomb. For most applications, however, the range of importance is narrower, changing from about 10 μm for micro-cellular structures to about 1–2 mm for foams on molten glass [1]. Size and structure of foams affect the density, thermal conductivity, Young’s modulus, and strength of the bulk material.

It is desirable to determine the properties of foams and cellular materials in real time to have better control of the formation process involved. Characterization of foam structures is best accomplished if non-intrusive diagnostic techniques are used; obviously, light is the prime candidate for such measurements. Optically, these systems are not necessarily thick, and their structure can be studied using inverse radiative analysis. For the food industry, such measurements will guarantee the consistency, the most important requirement; for biomedical applications, they will help in achieving the required precision. Several studies have been conducted to investigate the multiple light scattering in foams. Using such techniques Durian et al. [2] observed the transient behavior of coarsening in shaving cream. For an optically thick medium, the diffuse approximations have proven to yield accurate predictions of the angular distribution of diffusely transmitted light for liquid samples, such as colloidal suspensions and aqueous foams [3–5]. However, the change in polarization of incident light due to the structure of foams was not investigated in detail.

The cellular structures are likely to alter the polarization of the incident radiation. If we can relate the change in the physical characteristics of foamy layers to the changes in the polarization of angular reflection and transmission profiles, a series of smart sensors can be developed. For this purpose, we need to conduct careful experiments. First, however, it is desirable to theoretically understand the interaction of light with foams and predict the expected physical phenomena following a detailed computational effort.

In this work, the change in the degree of polarization of a collimated-polarized light incident on a one-dimensional foam layer containing air bubbles is studied using a combined Monte Carlo/Ray-Tracing approach. The medium temperature is assumed low; therefore, the radiative emission is neglected. Foam structures are modeled as air-laden spherical bubbles dispersed in a non-absorbing substrate. The separation distances between the mono- or polydispersed bubbles are assumed either to be constant or to follow a Gaussian distribution. The medium as well as the bubbles is considered non-absorbing; consequently, the depolarization ratios calculated correspond to maximum values.

In this study, we choose to use a geometric optics formulation rather than a more detailed physical optics analysis. The wavelength of the light is assumed much smaller than the size of the air bubbles, which is a premise of the ray-tracing approach. In addition, the effects due to dependent scattering, interference and diffraction are neglected. Mackowski et al. [6] have shown that if the size parameter ($x = \pi D/\lambda$) is greater than 20, the electromagnetic wave (EMW) analysis and the geometric-optics-based radiative transfer (RT) analysis predict similar absorption and scattering features by spherical particles. This means that for visible light, the minimum bubble diameter we can safely consider is about 3 μm, which is smaller than the typical bubble diameter we face in most practical systems [1]. The neglect of the dependent scattering, on the other hand, is acceptable if the clearance between the bubbles is at least one-half the wavelength of the radiation [7]. This is a reasonable assumption, as long as the bubbles do not touch each other. As discussed below, we have considered a few cases for touching bubbles; clearly, these results can be improved by considering dependent scattering effects. However, such an approach will require a more detailed EMW analysis, which is the beyond the scope of the present work.
2. Forward Monte Carlo simulation

The physical model used here is shown in Fig. 1. In simulations, we consider the perpendicular ($W_\perp$) and parallel ($W_\parallel$) polarization components of radiation intensity separately for each photon. We assume that the incident light has a 99% degree of polarization and incident on the medium at the origin. The radius of the bubbles is denoted $C$ ($D_{\text{sph}} = 2C$), and the bubbles are separated by a distance, $S_{\text{sep}}$. Both radius and separation distance between bubbles can either be a constant or follow a distribution function, which is chosen as

$$Z(S_{\text{sep}}) = \frac{1}{(2\pi)^{1/2}\ln \sigma_g} \exp \left[ -\frac{1}{2} \left( \frac{\ln(S_{\text{sep}}/N_n)}{\ln \sigma_g} \right)^2 \right],$$

where the standard deviation and median are denoted as $\sigma_g$ and $N_n$, respectively.

The scattering direction is based on the bubble geometry and determined using a ray-tracing technique in each bubble coupled with the Fresnel reflection and transmission. Three refractive indices are considered in the simulations: that for the medium ($n_{\text{med}}$), for the surroundings ($n_{\text{surr}}$), and for the bubbles ($n_{\text{sph}}$). A Monte Carlo technique is used to simulate the propagation of photons inside the medium. The number of photons used in the Monte Carlo simulation is chosen to be $50 \times 10^6$ based on the computations obtained in [8,9]. The discussions of MC models are available in the literature [7–13] and the details of the present approach are outlined in [8].

3. Scattering direction

When a photon hits a bubble, the subsequent scattering direction of the photon depends on the angle between the direction of photon and the tangential plane at the point of interaction (Fig. 2). For
these calculations, we should consider the area of a circle, with radius $C$. The area of a differential annular ring centered at a radius $r$ is expressed as
\[ dA = 2\pi r \, dr. \] (2)

The cumulative probability distribution function, $P(r)$, for a photon hitting any radius $r$ on the circle is obtained as
\[ P(r) = \frac{\int_0^r dA}{A} = \frac{\int_0^r 2\pi r \, dr'}{\pi C^2} = \left( \frac{r}{C} \right)^2. \] (3)

Replacing $P(r)$ with a random number $R$ yields
\[ r = C \sqrt{R}. \] (4)

Also, the incident angle with respect to the normal of the tangential surface is determined as
\[ \theta_i = \sin^{-1} \sqrt{R_\theta}, \] (5)
where $R_\theta$ is a uniformly distributed random number. We next examine the case when the incoming photon hits the surface of a bubble from inside the sphere. The expression for the incident angle is exactly the same as Eq. (4) (see [8]), as determined using the ray-tracing approach. The incident angle is only useful in determining the transmitted angle and modifying the weight of the photons. We need an angle with respect to the direction of propagation of the photon to be able to determine the direction cosines.

If the photon is reflected, the polar angle with respect to the propagating direction is expressed for both cases as
\[ \theta_{\text{travel},r} = 180^\circ - 2\theta_i. \] (6)

If the photon is transmitted, the polar angle is determined as
\[ \theta_{\text{travel},t} = |\theta_i - \theta_t|, \] (7)
where $\theta_t$ is obtained from Snell’s law. The azimuthal angle for either case would be determined using that of the isotropic scattering, which is
\[ \phi = 2\pi R_\phi, \] (8)
where $R_\phi$ is a random number.

4. Reflection and transmission

Once the incident and transmitted angles are known, the reflectance ($R$) and transmittance ($T$) are obtained from the Fresnel equations. A random number is drawn to decide if the photon is reflected or transmitted:

If $R_{\text{draw}} \geq T \Rightarrow$ reflected,

If $R_{\text{draw}} < T \Rightarrow$ transmitted.
After the direction of the photon is determined, its weight is modified accordingly. We obtain the ratio of the perpendicular component to the parallel component as [8]

\[
\left( \frac{W_\perp}{W_\parallel} \right)^{\text{new}} = \left( \frac{W_\perp}{W_\parallel} \right)^{\text{old}} \left( \frac{B_\perp}{B_\parallel} \right),
\]

where \( B \) is either transmission (\( T \)) or reflection (\( R \)).

5. The distance traveled inside the sphere

Once a photon enters a bubble the distance traveled is calculated using a ray-tracing approach based on the radius of the bubble. The distance of interaction inside a bubble is derived as [8]

\[
S_{\text{sph}} = D_{\text{sph}} \sin \theta,
\]

where \( \theta \) can be obtained from one of the following two cases:

- If the photon transmits into the sphere \( \Rightarrow \theta = 90^\circ - \theta_t \),
- If the photon reflects inside the sphere \( \Rightarrow \theta = 90^\circ - \theta_r \).

Eq. (9) asserts that the radius and the scattering direction of the photon predefined the distance traveled inside the sphere.

6. Mismatched boundary between the surroundings and medium

6.1. Specular reflection

A boundary is defined as “mismatched” if refractive indices are different on the sides of the interface. Under these conditions, specular reflection occurs when the laser beam first hits the interface. If the reflection measurements are carried out in air, then the measured profiles include the contribution of this reflectance. In applications where a laser beam is inserted into a liquid suspension using a fiber-optic probe, the boundary is not mismatched, and these reflections do not need to be considered since we are dealing with light leaving the probe. (In this study, we are not considering the probe–medium interface as the mismatch boundary.) Specular reflectance and transmittance are calculated as outlined in [8]. If a boundary is “mismatched,” the magnitude of the weight of the photon is reduced by a factor of \( R \) amount, where \( R \) is the fraction of specular reflection at the boundary. Then, \( (1 - R) \) fraction is considered as the weight of photons that travel into the medium.

6.2. Fresnel reflections

If the refractive indices at the boundaries are mismatched, then Fresnel reflection occurs once a photon is incident on a boundary. This means \( R \) fraction of the current photon weight would be reflected inside the medium while \( T \) fraction of the weight is transmitted out of the medium. The
ratio of the perpendicular component to the parallel component of the photon weight is modified and the magnitude of the weight is reduced by $T$ amount, which yields

$$W_{\text{mag}}^{\text{new}} = W_{\text{mag}}^{\text{old}}(1 - T).$$  \hspace{1cm} (11)

Moreover, the total internal reflection occurs within the medium as the photon encounters a decrease in the refractive index along the propagating path. The critical angle ($\theta_{\text{cr}} = \sin^{-1}(n_i/n_t)$) dictates this phenomenon. For any incident angle greater than the critical angle, the photon is reflected regardless of the decisive random number.

7. Storage and conversion of the photon histories

For the sake of accounting the change in the state of polarization of the photon at each distance of interaction, we track the magnitude of the photon weight and the ratio of its two weight components. By knowing these two parameters, we define $W_\perp$ and $W_\parallel$ as

$$W_\parallel = \frac{W_{\text{mag}}}{(W_\perp/W_\parallel) + 1},$$  \hspace{1cm} (12)

$$W_\perp = W_{\text{mag}} - W_\parallel.$$

(13)

Once a photon exits the medium, its weights for perpendicular and parallel components as well as the direction of propagation are stored. Two arrays $W_\perp(r, \theta)$ and $W_\parallel(r, \theta)$ are set up to store these data, which are then converted to the distributions of the degree of polarization. The angular and radial degree of polarization components is defined as (see also [14])

$$V(\theta) = \frac{|W_\perp(\theta) - W_\parallel(\theta)|}{W_\perp(\theta) + W_\parallel(\theta)},$$  \hspace{1cm} (14)

$$V(r) = \frac{|W_\perp(r) - W_\parallel(r)|}{W_\perp(r) + W_\parallel(r)},$$

where

$$W_\perp(\theta) = \sum_r W_\perp(r, \theta), \quad W_\parallel(\theta) = \sum_r W_\parallel(r, \theta)$$  \hspace{1cm} (15)

and

$$W_\perp(r) = \sum_\theta W_\perp(r, \theta), \quad W_\parallel(r) = \sum_\theta W_\parallel(r, \theta).$$  \hspace{1cm} (16)

The radial step size $\Delta r$ and angular step size $\Delta \theta$ are assumed 1/20 mm and $\pi/60$ rad, respectively. Note that “depolarization” refers to a decrease in the value of the $V$ function.
Table 1
Size and separation distance distributions considered

<table>
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<tr>
<th>Cases</th>
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<td>7</td>
<td>10–40</td>
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</tbody>
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Fig. 3. The distributions used for different sizes of bubbles and the separation distance between the bubbles.

Fig. 4. The size distributions of bubbles for cases 4 and 5.

8. Results and discussions

8.1. Effect of size distribution of bubbles

To investigate the effect of the size distribution of bubbles on the degree of polarization of the light, we compute five cases with five different size distributions of bubbles while holding the rest of the properties of the medium constant (see Table 1). The procedures for accounting a given size distribution in a Monte Carlo simulation are given in [8]. The size distributions used for cases 1–4 are shown in Figs. 3 and 4. The separation distance distribution between the bubbles for cases 1–5 is plotted in Fig. 3. In the first set of comparisons, we assumed matched refractive indices between the surroundings and the medium, which are 1.33 (water). We assume the medium contains only the air bubbles (n<sub>sph</sub> = 1.0). The thickness of the medium is assumed to be 2 mm.
The angular and radial distributions of the degree of polarization for the reflection, $V_R$, are plotted in Figs. 5 and 6, respectively. The results suggest that smaller size bubbles depolarizes radiation more, and $V_R(\theta)$ remains almost uniform over the entire angular domain. When the medium contains larger bubbles, there is less probability that photons will be scattered compared to the case where there are many smaller bubbles. Consequently, photons maintain most of their polarization states if smaller bubbles coalesce to form larger bubbles, since there will be fewer interactions between the photons and bubbles within the medium.

The radial profile of depolarization, $V_R(r)$, is more dependent on the size of the bubbles, as seen in Fig. 6. Within the first half millimeter of distance from the origin, depolarization is almost the same for cases 1 and 2, as well as cases 4 and 5; beyond that cutoff location, there is a slight increase and then a decrease in the value of $V_R$. The effect of bubble size on the change in the degree of polarization for the transmission follows the same trend as that for the reflection (not shown). In general, the larger the sphere is, the lower the degree of depolarization.

Fig. 7 depicts the reflected degree of polarization at $r = 2$ and 4 mm as a function of the median of the size distributions. The reflected $V$ decreases as the radius of the bubbles decreases. Note that these results suggest that the bubble size can be determined from carefully conducted experiments simply by measuring the depolarization ratio of reflected intensities. However, we observe the slopes of the ratios at radial distances of 2 and 4 mm decrease when the radius of the bubbles increases beyond 400 µm. Since the sensitivity of $V$ is small beyond the bubble size of 300 µm, the depolarization ratio would not allow the detection of larger bubbles reliably.

8.2. Effect of separation distance between bubbles

In order to investigate the effect of the separation distance between bubbles on the degree of depolarization, we repeated the simulations for cases 4 and 5 with ten times smaller separation distances (called as cases 6 and 7). The results for the radial degree of depolarization distributions
for both reflection and transmission, $V_R(r)$ and $V_T(r)$, are shown in Figs. 8 and 9, respectively. It is obvious that, with decreasing separation distance between bubbles depolarization increases. This shows that the denser medium with more bubbles (medium with smaller separation distance between the bubbles) tends to depolarize radiation more. The sharp difference between the profiles depicted in Fig. 9 is interesting and potentially useful; it is possible to use the depolarization of transmitted intensity to monitor the variation of bubble density in a medium in real time. Fig. 10 depicts the transmission depolarization $V_T$ at $r = 2$ and $4$ mm as a function of bubble radius. No significant difference is observed between the results at these two locations when the mean separation distance is smaller than 100 µm, suggesting that measurements can be performed reliably at any $r$ distance.
The results of this figure suggest that $V_T$ is a strong function of the mean bubble radius when the mean separation distance is below 100 µm, which can be used for diagnostic purposes.

Another interesting trend is observed when the separation distances between the bubbles are varied from zero to distances equivalent to several bubble radii (see Fig. 11). In general, the depolarization increases with decreasing distance between the bubbles. This is expected, as the decrease in distance is equivalent to an increase in bubble density. For the zero-separation case, the angular profile of reflected degree of polarization, $V_R$, displays a different trend; instead of being uniform over the entire range, it increases as the polar angle change from $0^\circ$ to $180^\circ$. This is mainly due to the increase in number of Fresnel reflection and transmission events photons go through before exiting the medium.

8.3. Change of polarization as a function of medium thickness

Another factor that would alter the degree of polarization is the thickness of the medium. Intuitively, we expect that the thicker the medium, the more likely the reflected and transmitted light would depolarize. This effect is demonstrated in Fig. 12. A three-time increase in medium thickness yields the reflected polarization ratio to drop from about 0.981 to 0.965 at zero angles. The radial and angular transmitted depolarization profiles are reported in [8].

8.4. Effect of mismatched boundaries

In the previous sections, we considered the boundaries that have matched refractive indices and studied the depolarization effects due to the curvature. In this section, we focus on the mismatched boundaries. In our previous computations, we observed that the mismatched boundaries tend to reduce the reflection and transmission while increasing the absorption inside the medium [8,9]. Hence, we would expect the same trend for the degree of depolarization.
In order to quantify the effect of mismatched boundary conditions, we consider three cases; (1) \( n_{\text{med}} = 1.50 \) and \( n_{\text{surr}} = 1.00 \), (2) \( n_{\text{med}} = 1.33 \) and \( n_{\text{surr}} = 1.00 \), and (3) \( n_{\text{med}} = 1.33 \) and \( n_{\text{surr}} = 1.33 \). The first case represents the medium with a higher effect of mismatched boundary compared to the second case. The last case is the medium with matched boundary. The results for these various cases are depicted in Figs. 13 and 14.

For mismatched boundaries, the \( V_R \) profiles do not cover the entire polar angle domain due to total reflection. It is well known that when light is incident on a boundary where the outside index of refraction is smaller than that of inside, a total reflection occurs at angles beyond \( \theta = \arcsin(n_{\text{out}}/n_{\text{in}}) \). Therefore, no reflected light will be registered beyond this angle. For the first case, the critical angle is 0.73 rad and for the second it is 0.85 rad; as expected, the simulations cease at about these angular locations. Also note that the two cases with mismatched boundaries retain their original polarization states at zero degrees of the polar angle due to the specular reflection at the incident location.

Generally, the medium with smaller difference between the indices of refraction would depolarize the radiation less. This is evident from all the simulations made. The first medium has the steepest variation in \( V_R(\theta) \) since it has largest difference between the indices of refraction. The second medium has a higher degree of polarization compared to the third medium. Although the third medium has matched refractive indices at the boundaries, the difference between indices of refraction of the medium and the bubbles is greater than that of the second medium. That is the reason the third medium depolarizes radiation more than the second medium. Although not shown here, the transmission profiles depicted similar trends [8].

9. Conclusions

In this paper, the change in the degree of depolarization of a collimated, polarized incident light on foams is studied. It is observed that the denser media (more bubbles) tend to depolarize the incident light more. This expected result means that smaller bubbles, smaller separation distances
between the bubbles, and increased foam thickness have stronger depolarization effects. It is also observed that the incident light is depolarized more if there are larger differences between the indices of refraction of the medium, bubbles, and the surroundings. The degree of depolarization strongly correlates with foam physical properties. This suggests that depolarization profiles can be used as an effective diagnostic tool to monitor the change in foam characteristics. For example, bubble size distributions can be accurately estimated by measuring $V_R$ profiles at distances 1–2 mm away from the location of the incident beam.

Only bubbles with spherical shape were studied in this work. The MC/ray-tracing approach presented can be extended easily to account for different shaped bubbles (see [1]). The effects of such bubbles as well as the experimental verification of the theoretical model will be discussed in another study.

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