EE522 SPRING 2001

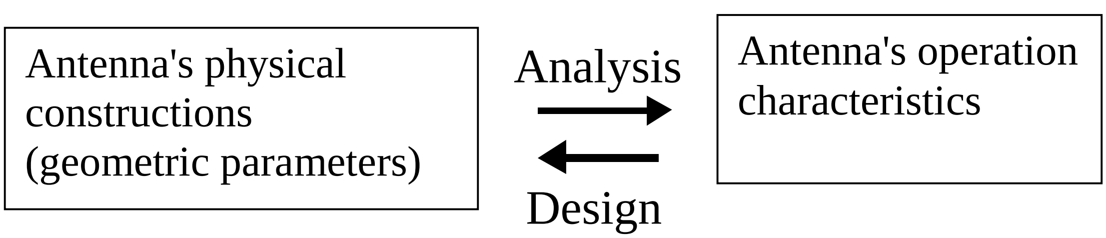
Course Web Site: http://www.engr.uky.edu/~cclu/ee522.html
Homework 1: (a) Use Eqn. (1.62) in (1.63) to derive (1.71a).
(b) Use (1.71a) in (1.60) to derive (1.71b).

- Antennas: used for transmitting and receiving signals
  Example: radar, communications

- Operation frequency: low, midium, high, very high, ultra high, microwave, …

- Construction: Printed, aperture, linear (wire), reflector,etc.

- Analysis and design:



- Major parameters of antennas:
  - Input impedance
  - Radiation pattern
  - Polarization
  - Efficiency
  - Gain

- Other specifications:
  - Size
  - Weight
  - Power capacity
  - Reliability
  - Shape

Antenna performance and operation frequency

1. **Electrically small antennas:**
   Electrical size is much smaller than a wavelength
   Very low directivity
   Low input resistance and high input reactance
   Low radiation efficiency
   Example: small dipole, small loop
   Application: Low frequency.

2. **Resonant Antennas:**
   Electrical size is comparable to a wavelength.
   Low to moderate gain
   Real input impedance (small reflection)
   Narrow bandwidth
   Example: Half-wave dipole, single microstrip patch
   Application: PCS communication

3. **Broadband antenna**
   Real input impedance
   Low to moderate gain
   Gain, impedance remain at wide frequency band.
   Example: Log-periodic array, sprial antenna.
   Application: Multipurpose communication.

3. **Aperture antennas:**
   Electric size much larger than wavelength
   High gain
   Moderate bandwidth
   Example: Horn, Reflector antennas.
   Application: Satellite signal receiving, Radar.

- **Typical Design cycles**

  (1) Parameter specification

  (2) Preliminary design

  (3) Detailed design using theoretical equations, empiric formulas, and computer programs.

  (4) If output parameters are satisfactory, done. Else go to (3) if more iterations are needed. Or re-specify parameter and go to (2).

Important: knowledge of antenna theory
Working experience
Knowledge in using computer programs

Example:
Design a satellite TV signal receiving antenna that has directivity of more than 32dB at 12GHz.
Preliminary design: reflector antenna, circular shape.
Theoretical equation for directivity:

$$D = \frac{4\pi A^2}{\lambda^2} = \frac{4\pi\left(\pi a^2\right)}{\lambda^2}$$

32dB ---- D=1585, 12GHz---- $\lambda = 0.3/12 = 0.083m$

$$a = \sqrt{\frac{D\lambda^2}{4\pi^2}} = \frac{\lambda}{2\pi}\sqrt{D} = \frac{0.083}{2\times 3.14159}\sqrt{1585} = 0.526m$$

- **Maxwell's equation in frequency domain**

Time factor: $e^{j\omega t}$

Time domain field = Re{Frequency domain field X $e^{j\omega t}$ }

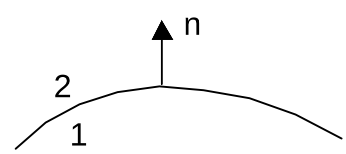
$$\nabla\times\bar{E}(\bar{r}) = j\omega\mu\bar{H}(\bar{r}) - \bar{M}$$
$$\nabla\times\bar{H}(\bar{r}) = -j\omega\varepsilon\bar{E}(\bar{r}) + \bar{J}$$
$$\nabla\cdot\varepsilon\bar{E}(\bar{r}) = \rho(\bar{r})$$
$$\nabla\cdot\mu\bar{H}(\bar{r}) = 0$$

$\bar{M}$: Equivalent magnetic current (does not physically exist)

$\bar{J}$: Electric current (applied or equivalent)

- **Boundary condition: General**

$$\hat{n}\times\left(\bar{E}_2 - \bar{E}_1\right) = -\bar{M}$$
$$\hat{n}\times\left(\bar{H}_2 - \bar{H}_1\right) = \bar{J}$$



- Boundary condition: conducting interface

$\bar{E}_{\tan} = 0, \qquad \bar{H}_{\tan} = \bar{J}_s$

- Radiation solution

Vector potential: $\bar{H} = \frac{1}{\mu} \nabla \times \bar{A}$

Scalar potential: Φ : $\bar{E} + j\omega\bar{A} = -\nabla\Phi$

Lorentz gauge: $\nabla \cdot \bar{A} = -j\omega\varepsilon\mu\Phi$

Equation for $\bar{A}$:

$$\nabla^2 \bar{A} + k^2 \bar{A} = -\mu \bar{J}$$

Solution:

$$\bar{A}(\bar{r}) = \mu \int_S \frac{e^{-jkR}}{4\pi R} \bar{J}(\bar{r}\,')d\bar{r}\,', \qquad R = |\bar{r} - \bar{r}\,'|$$

$$\bar{H} = \frac{1}{\mu} \nabla \times \bar{A}$$

$$\bar{E} = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}, \qquad \text{in source-free region.}$$

Key: The current distribution on the antenna surface.

- **Ideal Dipole:**

Infinitesimal element of current
Length $\Delta z << \lambda$
Current density: I =constant
Orientation: $\hat{z}$
Location: origin

Vector potential:

$$\bar{A}(\bar{r}) = \mu \int_{-\Delta/2}^{\Delta/2} \frac{e^{-j\beta R}}{4\pi R} \left(\hat{z} I\right) dz'$$

$$= \hat{z} \frac{\mu I \Delta z e^{-j\beta r}}{4\pi r} \operatorname{sinc}\left(\frac{1}{2} \beta \Delta z \cos\theta \right)$$

Magnetic field: $\bar{H}(\bar{r}) = \frac{1}{\mu} \nabla \times \bar{A} = \nabla \times \left(\hat{z} I \Delta z \frac{e^{-j\beta r}}{4\pi r} \right)$

Using a vector identity: $\nabla \times \left(\phi \bar{B} \right) = \nabla \phi \times \bar{B} + \underbrace{\phi \nabla \times \bar{B}}_{=0}$

$$\bar{H} = \nabla \left(\Delta z I \frac{e^{-j\beta r}}{4\pi r} \right) \times \hat{z} = \frac{I \Delta z}{4\pi} \frac{\partial}{\partial r} \left(\frac{e^{-j\beta r}}{r} \right) \times \hat{z}$$

$$= \frac{I \Delta z}{4\pi} j\beta \left(1 + \frac{1}{j\beta r} \right) \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\phi}$$

$$\bar{E} = \frac{I \Delta z}{4\pi} j\omega\mu \left[1 + \frac{1}{j\beta r} + \frac{1}{\left(j\beta r \right)^2} \right] \frac{e^{-j\beta r}}{r} \sin\theta \hat{\theta}$$

$$+ \frac{I \Delta z}{2\pi} j\omega\mu \left[\frac{1}{j\beta r} + \frac{1}{\left(j\beta r \right)^2} \right] \frac{e^{-j\beta r}}{r} \cos\theta \hat{r}$$

Three regions:

(i) Far-field region (radiation region): $\beta r >> 1$

$$\bar{E} = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-j\beta r}}{r} \sin\theta\, \hat{\theta} \qquad \bar{H} = \frac{I\Delta z}{4\pi} j\beta \frac{e^{-j\beta r}}{r} \sin\theta\, \hat{\phi}$$

Important properties:

(1) Field amplitude $\propto \frac{1}{r}$

(2) Fields do not have $\hat{r}$ component

(3) $\bar{E}, \bar{H}, \hat{r}$ form right-hand-rule and

$$\bar{E} = \eta \bar{H} \times \hat{r}, \quad E / H = \sqrt{\varepsilon / \mu} = \eta$$

(ii) Intermediate region: $\beta r \approx 1$

(iii) Near-field region: $\beta r << 1$

$$\bar{E} = \frac{I\Delta z}{4\pi} j\omega\mu \frac{e^{-j\beta r}}{(j\beta)^2 r^3} \left(\hat{\theta} \sin\theta + 2\hat{r} \cos\theta\right)$$

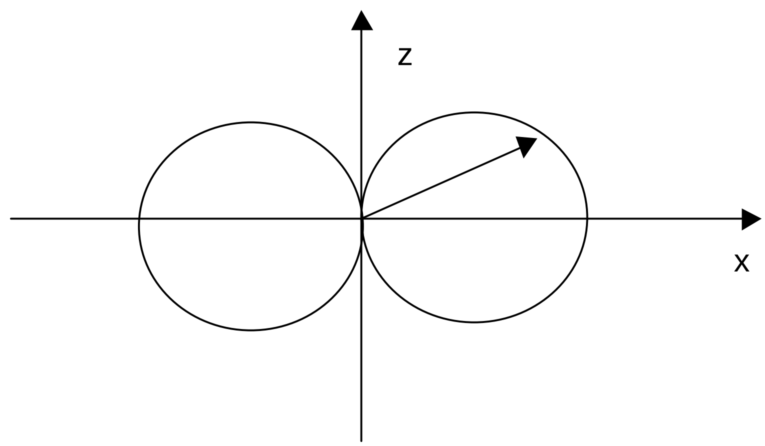
** Angular pattern is the same as an electrostatic dipole.

- **Radiation pattern:**

  Angular variation of electric field amplitude or phase at far-field region. It is usually normalized so that the maximum value is 1. In the above ideal dipole example, the radiation pattern is

$$F(\theta,\phi)=\sin\theta$$

Graphical representation: often plot in polar-coordinate system.



- **Far-field calculation**

Far-field approximation:	$R \approx r$ for amplitude factors. $R \approx r - \hat{r} \cdot \overline{r}\,'$ for phase factors.

Under far-field approximation, the radiation integrals to calculate radiate fields can be simplified.

The potential integral now become

$$\overline{A} = \mu \int_S \frac{e^{-j\beta(r - \hat{r}\cdot\overline{r}\,')}}{4\pi r} \overline{J}(\overline{r}\,') d\overline{r}\,' = \frac{\mu e^{-j\beta r}}{4\pi r} \int_S e^{j\beta \hat{r}\cdot\overline{r}\,'} \overline{J}(\overline{r}\,') d\overline{r}\,'$$

$\overline{E} = -j\omega \overline{A}$ (for $\hat{\theta}$ and $\hat{\phi}$ components)

That is: $E_\theta = -j\omega \hat{\theta} \cdot \overline{A}$, $\quad E_\phi = -j\omega \hat{\phi} \cdot \overline{A}$

The radiation patterns:

$$\begin{bmatrix} F_\theta(\theta,\phi) \\ F_\phi(\theta,\phi) \end{bmatrix} = \frac{-j\omega\mu}{4\pi} \int_S e^{j\beta \hat{r}\cdot\overline{r}\,'} \begin{bmatrix} \hat{\theta} \cdot \overline{J} \\ \hat{\phi} \cdot \overline{J} \end{bmatrix} d\overline{r}\,'$$

- **Far-field distance r_{ff} :**

  The distance where the phase error is $\pi/8 = 22.5^o$

$$R = |\bar{r} - \bar{r}'| = r - \hat{r}\cdot\bar{r}' + \frac{1}{2}\left(\frac{r'}{r}\right)^2 \left[1 - (\hat{r}\cdot\hat{r}')^2\right] + \cdots$$

Phase error $\leq \frac{1}{2}\left(\frac{r'}{r}\right)^2 \beta$

If D is the diameter of the antenna volume (defined as the diameter of the smallest sphere that encloses the antenna), then $r' \leq D/2$, hence

$$\frac{1}{2}\left(\frac{D/2}{r_{ff}}\right)^2 \frac{2\pi}{\lambda} = \frac{\pi}{8}, \qquad r_{ff} = \frac{2D^2}{\lambda}$$

The distance calculated above is usually used as a minimum distance criteria for antenna measurement. For example, to measure an antenna of diameter 1m at frequency of 10GHz, the minimum distance (distance between transmission and receiving antennas) should be $2\times 1^2 / 0.03 = 66.7m$

- **Steps to obtain radiation field:**

(1) Find $\bar{A}(\bar{r}) = \frac{\mu e^{-j\beta r}}{4\pi r}\int_S e^{-j\beta\hat{r}\cdot\bar{r}'}\bar{J}(\bar{r}')d\bar{r}'$, where the integration is over the antenna surface (for linear antennas, the integration becomes line integral).

(2) Find $\bar{E}(\bar{r}) = -j\omega\bar{A}$ (for $\hat{\theta}$ and $\hat{\phi}$ components)

(3) Find $\bar{H}(\bar{r}) = \hat{r}\times\bar{E}/\eta$

Example (ideal dipole)

$$\bar{A} = \frac{\mu e^{-j\beta r}}{4\pi r}\int_{-\Delta z/2}^{\Delta z/2} e^{-j\beta\hat{r}\cdot\bar{r}'} I\hat{z}\,dz' = \frac{\mu I\Delta z}{4\pi}\frac{e^{-j\beta r}}{r}\left(\hat{r}\cos\theta + \hat{\theta}\sin\theta\right)$$

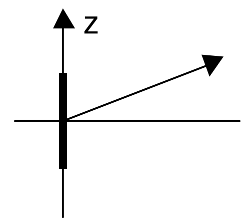
$$\bar{E} = j\omega\mu\frac{I\Delta z}{4\pi}\frac{e^{-j\beta r}}{r}\sin\theta\,\hat{\theta}$$

$$\bar{H} = \hat{r}\times\bar{E}/\eta = \frac{I\Delta z}{4\pi}\frac{e^{-j\beta r}}{r}j\beta\sin\theta\,\hat{\phi}$$

(note: $\eta = \omega\mu/\beta$)

- **Uniform Line Source**

Length: L
Current density:
Uniform I_0
Location: -L/2 to L/2
Orientation: $\hat{z}$



Potential:

$$\bar{A} = \frac{\mu e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} e^{j\beta \hat{r}\cdot\bar{r}'} I_0 \hat{z}\, dz'$$

$$= \hat{z} \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \int_{-L/2}^{L/2} e^{j\beta z'\cos\theta} dz'$$

$$= \hat{z} \frac{\mu I_0 e^{-j\beta r}}{4\pi r} \frac{e^{j\beta(L/2)\cos\theta} - e^{-j\beta(L/2)\cos\theta}}{j\beta\cos\theta}$$

$$= \hat{z} \frac{\mu I_0 L e^{-j\beta r}}{4\pi r} \frac{\sin[\beta(L/2)\cos\theta]}{\beta(L/2)\cos\theta}$$

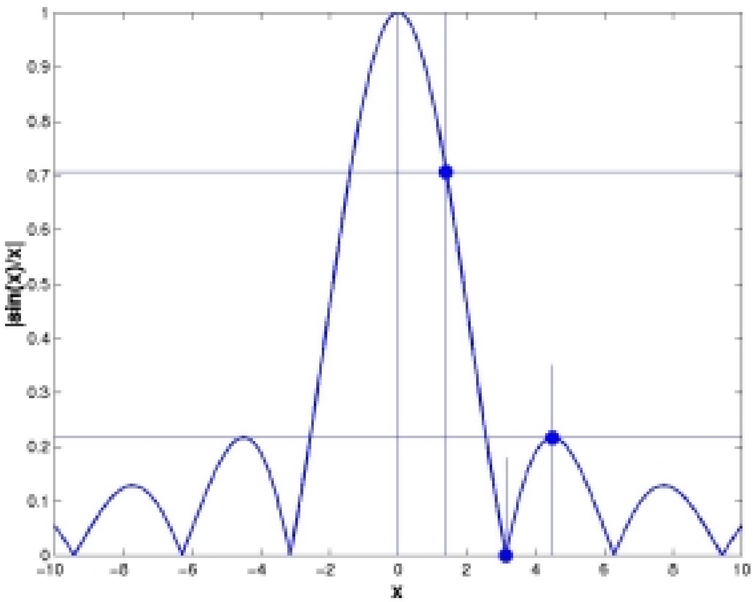
since $\hat{z} = \hat{r}\cos\theta + \hat{\theta}\sin\theta$

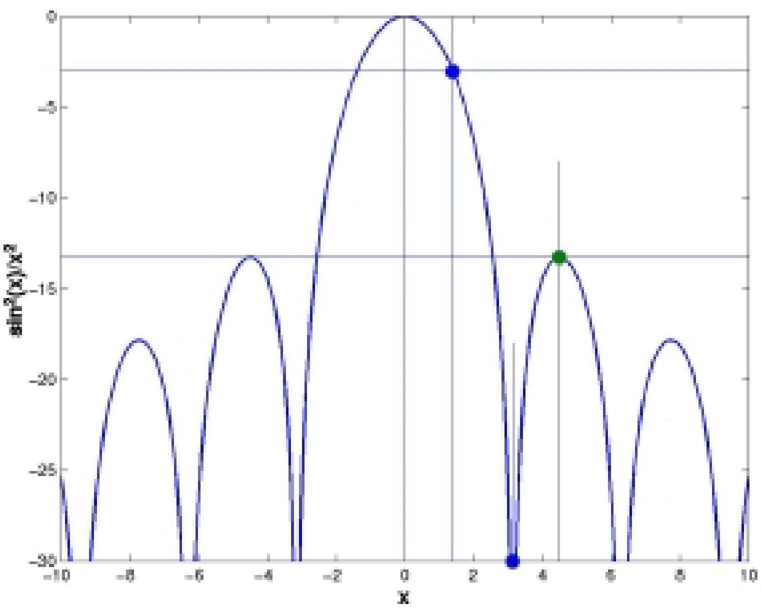
$$\bar{E} = \hat{\theta}\, j\omega\mu \frac{I_0 L e^{-j\beta r}}{4\pi r} \sin\theta \frac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}$$

Radiation pattern: $F_\theta(\theta,\phi) = \sin\theta \dfrac{\sin[(\beta L/2)\cos\theta]}{(\beta L/2)\cos\theta}$

Maximum radiation directon: $\theta = \pi/2$.

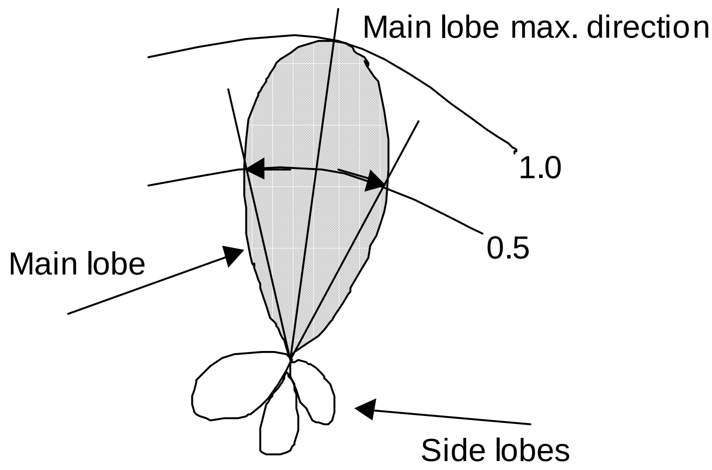
- **Properties of "sinc()" funtion:** $\sin(x)/x$





(1) Maximum: x=0: level=1 (0 dB)
(2) Half power point at x=1.3916 (Level=-3dB)
(3) Zero-pointts at $x = n\pi$ **, n=integers**
(4) First side lobe: x=4.493, Level=0.217 (-13.2dB)
(5) Second side lobe: x=7.725, Level=0.128 (-17.8dB)

- **Radiation pattern parameters:**



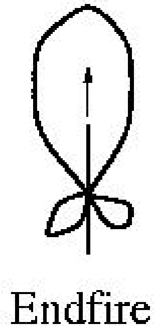
Side lobe level (SLL) is defined as:

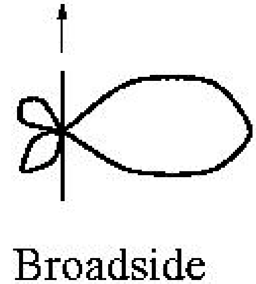
$$SLL_{dB} = 20\log\frac{|F(SLL)|}{|F(\max)|}$$

Half-power bandwidth: $HP = |\theta_{HP,left} - \theta_{HP,right}|$

Main lobe direction vs. antenna orientation:

Endfire: Main lobe parallel to antenna axis.
Broadside: Main lobe perpendicular to antenna axis





Use MATLAB function "fzero" to find roots

For example, to find the half power point of the pattern $\sin(2\sin\theta)/(2\sin\theta)$**, we need to solve the following equation:**

$$\left|\sin(2\sin\theta_h)/\left(2\sin\theta_h\right)\right| = 0.707, \quad \theta_h = ?$$

In MATLAB prompt ">", type

```
fzero('abs(sin(2*sin(x))/(2*sin(x)))-0.707',[0.01,2])
```

Will get

x=0.7697

You can plot the function to estimate the initial solution region.