EE522 NOTES (2)

- **Radiation rasistance R_r is defined by**

Radiation power: $P = \frac{1}{2} R_r \, |I_A|^2$

Calculation of radiation power:

$$P = \int_0^{2\pi} \int_0^{\pi} \frac{1}{2\eta} \left| r\bar{E} \right|^2 \sin\theta \, d\theta d\phi$$

For ideal dipole: $R_r = 80\pi^2 \left(\frac{\Delta z}{\lambda} \right)^2$ (Ω)

- **Directivity**

(1) To describe how much an antenna concentrates the energy as a function of directions.
Isotropic antenna: radiates energy with the same intensity for all directions.
High concentration: focus energy in a specific direction with very narrow beamwidth (e.g., some radar antenna).

(2) It is described using relative **radiation intensity.**

(3) Radiation intensity**:** $U(\theta, \phi) = \frac{1}{2\eta} \left| r\bar{E} \right|^2$ (Watts/Angle)

where: $\bar{E}$ is the far-field electric field (in V/m)
r is the distance (in far-field region)
η is the wave impedance in Ohm.

Radiation power: $$P = \int_{\Omega} U(\theta,\phi)\sin\theta d\theta d\phi$$

Average radiation intensity: $$U_{ave} = \frac{P}{4\pi}$$

Maximum radiation intensity: $U_m = \max U(\theta,\phi)$

Example: find the U, U_{ave}, U_m for the ideal dipole:

Solution: For ideal dipole, the radiation field is

$$\bar{E} = \hat{\theta}\left(j\omega\mu \frac{I\Delta z}{4\pi}\right)\cdot \frac{e^{j\beta r}}{r}\cdot \sin\theta$$

Hence

$$U(\theta,\phi) = \frac{1}{2\eta}\left| r\cdot\left(j\omega\mu \frac{I\Delta z}{4\pi}\right)\cdot \frac{e^{j\beta r}}{r}\cdot \sin\theta\right|^2 = \frac{(\omega\mu I\Delta z)^2}{8\eta\pi^2}\sin^2\theta$$

$$U_m = \frac{(\omega\mu I\Delta z)^2}{8\eta\pi^2}$$

$$U_{ave} = U_m \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta d\phi = \frac{2}{3}U_m$$

It can be shown that: $$U(\theta,\phi) = U_m \left|\bar{F}(\theta,\phi)\right|^2$$

Directivity: ration of radiation intensity over average radiation intensity.

$$D(\theta,\phi)=\frac{U(\theta,\phi)}{U_{ave}}, \qquad D=\frac{U_m}{U_{ave}}$$

Note: (1) It is usually given in terms of dB such that

$$D_{dB}=10\log(D)$$

(2) D can be considered as the ratio of the radiation intensities of the antenna with an isotropic antenna if the two antennas have the same radiation power.

For the ideal dipole, its directivity is

$$D_{\text{dipole}}=\frac{U_m}{U_{ave}}=\frac{U_m}{2U_m/3}=1.5 \quad (\text{or } 1.76\text{ dB}).$$

Example: calculate the directivity of an antenna whose radiation pattern is given as

$$F_\theta(\theta)=\begin{cases}1, & 0<\theta<\pi/2\\ 0, & \text{Elsewhere}\end{cases}, \qquad F_\phi=0$$

Solution: Let $U(\theta,\phi)=U_m\left|\bar{F}_\theta\right|^2$, then

$$U_{ave}=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}U_m\left|F_\theta\right|^2\sin\theta\ d\theta d\phi$$

$$=(U_m/2)\int_0^{\pi/2}\sin\theta\, d\theta$$

$$=U_m/2$$

$$D(\theta,\phi)=\frac{U_m\left|F_\theta(\theta)\right|^2}{U_m/2}=2\left|F_\theta(\theta)\right|^2$$

$$D=2$$

Gain

Practical antennas are made of conducting materials that have finite conductivity (or non-zero surface resistance). Hence part of the power input to an antenna is dissipated by the antenna, and the rest is radiated into space. Consider two antennas:

(1) The actually antenna, the radiation intensity is $U(\theta,\phi)$.

(2) A lossless antenna that has isotropic radiation pattern, hence its radiation intensity is $U_0 = P_{in} / 4\pi$

The antenna gain is defined as the radio of the above two:

$$G(\theta,\phi) = \frac{U(\theta,\phi)}{P_{in}/(4\pi)}, \qquad G = \frac{U}{P_{in}/(4\pi)}$$

It is also usually given in terms of dB.

Because of the power dissipation by the antenna, the radiation power is usually smaller than the input power. The ratio of the radiation power over the input power is defined as the **radiation efficiency**:

$$e_r = \frac{P}{P_{\text{in}}}, \qquad (\ 0 \le e_r \le 1\ )$$

Note: Comparing the definition of the directivity and the gain, one can see that since $P \le P_{\text{in}}$, we have $G \le D$. In fact, the radiation efficiency can be written as $e_r = G / D$.

- **Input impedance:** $Z_A = R_A + jZ_A$

  R_A**: Associated with ohmic loss and radiation.**

  Z_A**: Associated with power stored in near-field.**

Consider R_A: since $P_{in} = \frac{1}{2} R_A |I_A|^2 = P + P_{ohmic}$, we can show that

$$R_A = R_r + R_{ohmic}$$

R_r stands for the radiation resistance: $P = \frac{1}{2} R_r |I_A|^2$

R_{ohmic} stands for ohmic resistance: $P_{ohmic} = \frac{1}{2} R_{ohmic} |I_A|^2$

For wire antennas, the ohmic resistance is related to the conductivity of the wire with uniform current by:

$$R_{ohmic} = \begin{cases} \frac{L}{2\pi a} R_s, & \text{Uniform current} \\ \frac{L}{2\pi a} \frac{R_s}{3}, & \text{Triangle current} \end{cases}, \qquad R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

Since the radiation power $P = \frac{1}{2} R_r |I_A|^2$

The input power $P_{in} = \frac{1}{2} R_{in} |I_A|^2$, we have

Radiation efficiency: $e_r = \frac{P}{P_{in}} = \frac{R_r}{R_A} = \frac{R_r}{R_r + R_{ohmic}}$

Note:

(1) An antenna is said to be resonant if $Z_A = 0$.

(2) Effect of impedance mismatch: power reflection.

The reflection coefficient: $\Gamma = (Z_A - Z_c)/(Z_A + Z_c)$

(1) The effects of ohmic loss: reduce radiation efficiency and introduces signal noise.

Example: calculate the radiation efficiency of an AM car radio antenna.

Solution:

This antenna is a monopole over ground. It can be approximated by a dipole in free-space.

Calculation:

$$\omega = 2\pi \times 10^{6} \text{ (Radians)}$$
$$\mu = 4\pi \times 10^{-7} \text{ (Farady/m)}$$
$$\sigma = 2\times 10^{6} \text{ (Sigma/m) for steel.}$$

$$L = 2\times 31(inch) = 62\, inch = 1.575m$$
$$a = (1/8)inch = 0.159cm$$
$$f = 300MHz \quad (\lambda = 300m >> 1.57m)$$
$$R_r = 80\pi^2 \left(1.57/300\right)^2 = 0.00545\Omega$$
$$R_s = \sqrt{\left(2\pi \times 10^6 \times 4\pi \times 10^{-7}\right) / \left(2\times 2\times 10^6\right)} = 0.0014\Omega$$
$$R_{ohmic} = \frac{1.575}{2\times\pi\times 0.159}\ \frac{0.0014}{3} = 0.0736\Omega$$
$$e_r = \frac{R_r}{R_r + R_{ohmic}} = \frac{0.0054}{0.0054 + 0.0736} = 6.7\%$$

Antenna Polarization: Trace of the instantaneous electric field.

(1) Linear polarization
(2) Circular polarization (LHCP and RHCP)
(3) Elliptical polarization (LHEP and RHEP)

Axial ratio: $\text{AR} = \dfrac{E_{\max}}{E_{\min}}$,

$$\begin{cases} \text{AR} = \infty, & \text{for linear polarization} \\ \text{AR} = 1, & \text{for circular polarzation} \\ 0 < \text{AR} < \infty, & \text{for ellipticcal polarzatoin.} \end{cases}$$

Vertical and Horizontal polarization (linear) in spherical coordinate system:

Vertical polarization: E is linear and in $\hat{\theta}$ direction.

Horizontal polarization: E is linear and in $\hat{\phi}$ direction.