EE522 NOTES (4)

Small loop (Loop dimension << Wavelength)

Two methods to get radiation field:

(1) Using Duality theorem

In the solution for a small dipole, replace

$I\Delta z$	by	$I^m\Delta z$
μ	by	ε
ε	by	μ
$\bar{E}$	by	$-\bar{H}$
$\bar{H}$	by	$\bar{E}$

The radiation field by a small dipole is

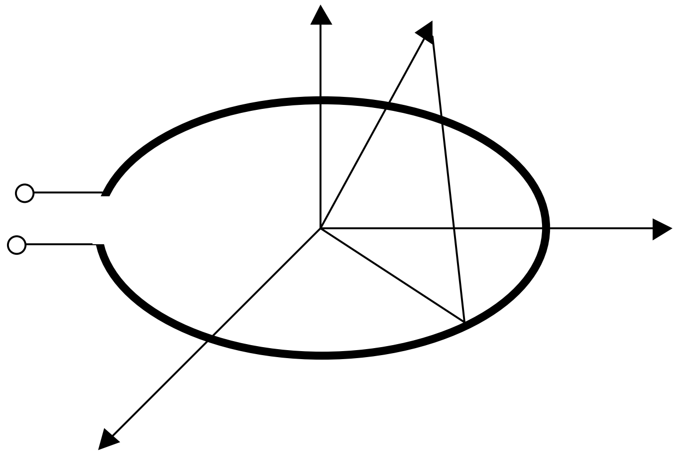
$$\bar{E} = \hat{\theta}\frac{j\omega\mu I\Delta z}{4\pi}\frac{e^{-j\beta r}}{r}\sin\theta$$

Hence the magnetic field radiated by a small loop is

$$\bar{H} = \hat{\theta}\frac{j\omega\varepsilon I^m\Delta z}{4\pi}\frac{e^{-j\beta r}}{r}\sin\theta$$

In the above, the factor $I^m\Delta z$ is the magnetic dipole moment.

(2) Potential integration



$$\bar{A} = \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \oint e^{j\beta \hat{r}\cdot\bar{r}'} \bar{J} dl'$$

$$dl' = ad\phi'$$

$$\bar{J} = I\hat{l}' = Ia\hat{\phi}' = Ia(-\hat{x}\sin\phi' + \hat{y}\cos\phi')$$

$$\bar{r}' = a(\hat{x}\cos\phi' + \hat{y}\sin\phi')$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\hat{r}\cdot\bar{r}' = a\left[\sin\theta\cos\phi\cos\phi' + \sin\theta\sin\phi\sin\phi'\right] = a\sin\theta\cos(\phi' - \phi)$$

$$\bar{A} = \frac{\mu e^{-j\beta r}}{4\pi r}(Ia^2)\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}(-\hat{x}\sin\phi' + \hat{y}\cos\phi')d\phi'$$

$$E_\theta = \frac{j\omega\mu Ia^2 e^{-j\beta r}}{4\pi r}\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}(\cos\theta\cos\phi\sin\phi' - \cos\theta\sin\phi\cos\phi')\,d\phi'$$

$$= \frac{j\mu(Ia^2)e^{-j\beta r}}{4\pi r}\cos\theta\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}\sin(\phi - \phi')\,d\phi'$$

$$= 0$$

$$E_\phi = \frac{-j\omega\mu(Ia^2)e^{-j\beta r}}{4\pi r}\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}(\sin\phi\sin\phi'+\cos\phi\cos\phi')\,d\phi'$$

$$= \frac{-j\omega\mu(Ia^2)e^{-j\beta r}}{4\pi r}\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}\cos(\phi'-\phi)\,d\phi'$$

$$= \frac{-j\omega\mu Ia^2 e^{-j\beta r}}{4\pi r}\int_0^{2\pi} e^{j\beta a\sin\theta\cos(\phi'-\phi)}\cos(\phi'-\phi)\,d\phi'$$

$$= -\frac{j\omega\mu Ia^2 e^{-j\beta r}}{4\pi r}2\pi j\,J_1(\beta a\sin\theta)$$

$$= I\eta\beta^2 S\frac{e^{-j\beta r}}{4\pi r}\sin\theta \qquad (\beta a \ll 1)$$

$$\bar{H} = \frac{1}{\eta}\hat{r}\times\left(\hat{\phi}E_\phi\right) = -\hat{\theta}\,\beta^2 S\frac{e^{-j\beta r}}{4\pi r}\sin\theta, \qquad (\beta a \ll 1)$$

Radiation intensity: $U(\theta,\phi) = U_m\sin^2\theta$

Average Radiation intensity: $U_{ave} = 1.5U_m$

Directivity: $D = 1.5$

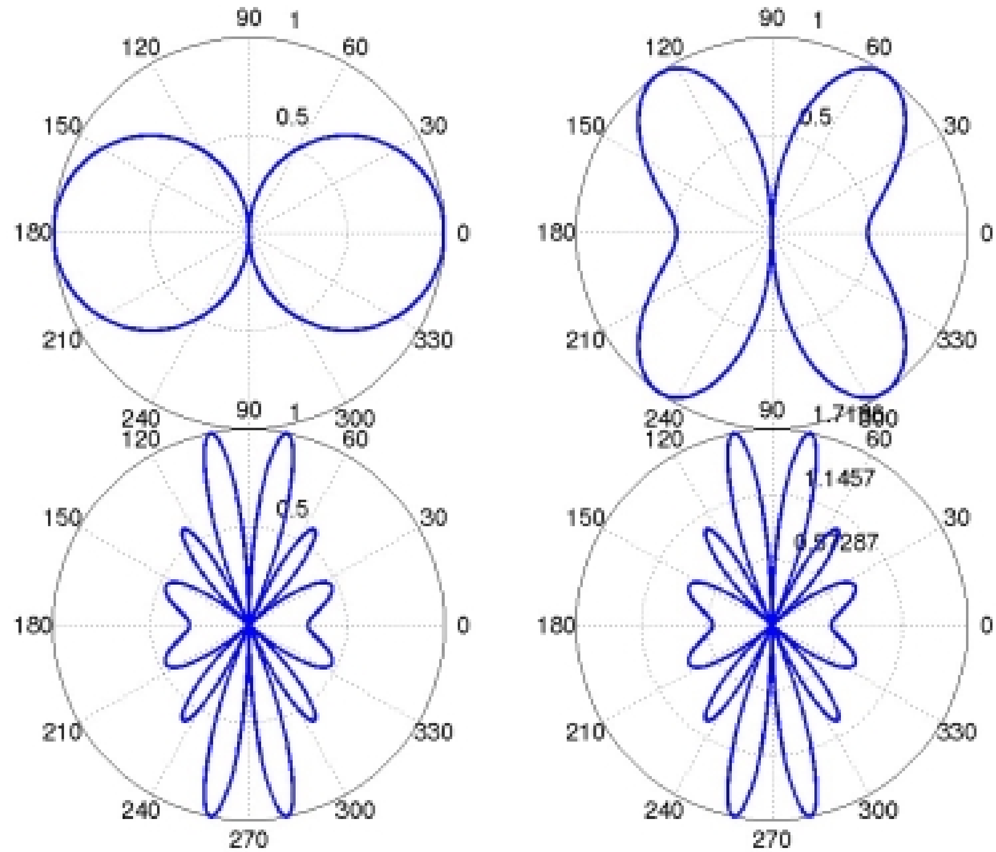
Radiation resistance: $R_r = 31200(S/\lambda^2)^2$

If the number of turns is N, then the radiation resistance is

$$R_r = 31200(NS/\lambda^2)^2$$

If N-turn of current is wound on a ferrite core forming a **loop-stick antenna,** and the relative permeability of the ferrite core is μ_{eff} , then the radiation resistance is

$$R_r = 31200(\mu_{\text{eff}}NS/\lambda^2)^2$$



a=1.5λ

a=2λ