EE522 NOTES (6)
NUMERICAL METHOD FOR WIRE ANTENNAS

1. Integral equation

Electric field generated by current distribution:

$$E_z = \frac{1}{j\omega\varepsilon}\int_0^L \left(\frac{\partial^2 g}{\partial z^2} + \beta^2 g \right) I(z')dz', \qquad (1)$$

Electric field generated by excitation: E_z^i

Boundary condition on conducting surface of wire:

$$E_z + E_z^i = 0, \qquad z \in [0, L], \qquad (2)$$

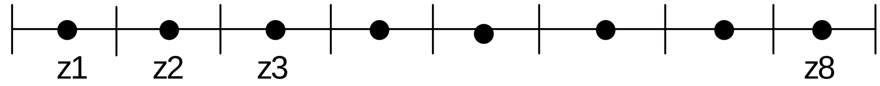
Use (1) in (2):

$$\frac{1}{j\omega\varepsilon}\int_0^L \left(\frac{\partial^2 g}{\partial z^2} + \beta^2 g \right) I(z')dz' = -E_z^i, \qquad z \in [0, L]$$

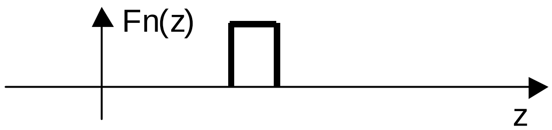
This is the integral equation for the unknown current distribution $I(z')$. Where the Green's function g(z,z') is

$$g(\bar{r}, \bar{r}') = \frac{e^{-j\beta R}}{4\pi R}, \qquad R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

2. Divide the antenna into N-segments, the center of the n-th segment is denoted as z_n:

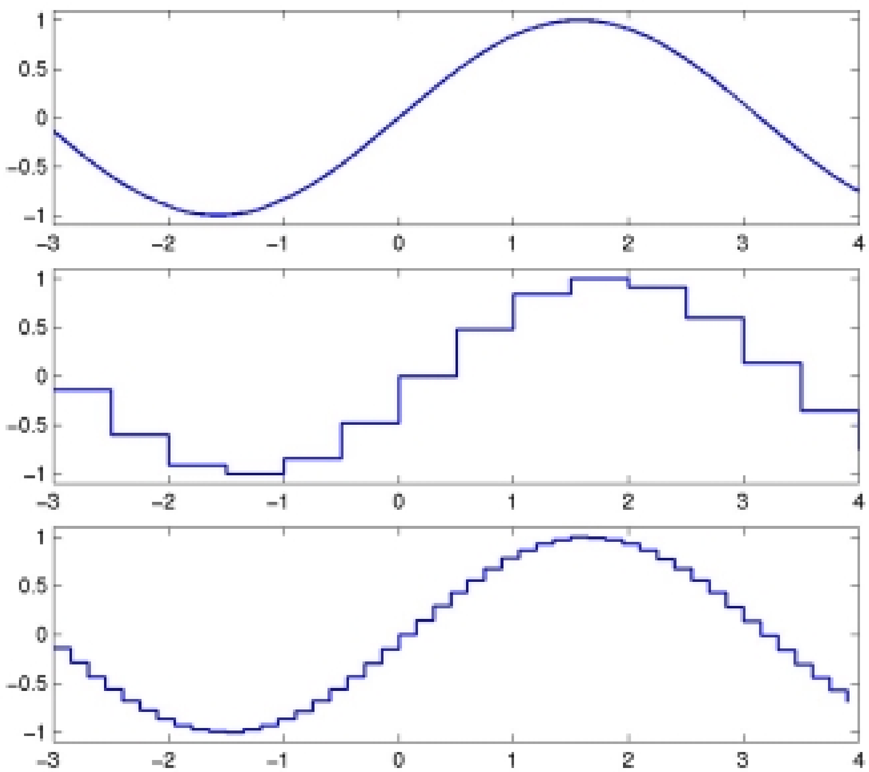


3. Approximate representation of the current I(z') in terms of pulse functions:



$$F_n(z) = \begin{cases} 1, & |z - z_n| < \Delta/2 \\ 0, & \text{Elsewhere} \end{cases}$$

$$I(z') = \sum_{n=1}^{N} I_n F_n(z')$$



The integral equation now becomes

$$\sum_{n=1}^{N} I_n \left[\frac{1}{j\omega\varepsilon} \int_0^L \left(\frac{\partial^2 g}{\partial z^2} + \beta^2 g \right) F_n(z') dz' \right] = -E_z^i, \quad z \in [0, L]$$

Let $f(z, z_n) = \frac{1}{j\omega\varepsilon} \int_0^L \left(\frac{\partial^2 g}{\partial z^2} + \beta^2 g \right) F_n(z') dz'$, we have

$$\sum_{n=1}^{N} I_n f(z, z_n) = -E_z^i, \quad z \in [0, L]$$

(3) Point matching: Let the above equation valid for

$z = z_1, z_2, \cdots, z_N$, we get

$$\sum_{n=1}^{N} I_n f(z_m, z_n) = -E_z^i(z_m), \quad m = 1, 2, \cdots, N$$

Matrix element:

$$f(z_m, z_n) = \frac{1}{j\omega\varepsilon} \int_{z_n-\Delta/2}^{z_n+\Delta/2} \frac{e^{-j\beta R}}{4\pi R^5} \left[(1 + j\beta R)(2R^2 - 3a^2) + \beta^2 a^2 R^2 \right] dz'$$

(4) Solution of the matrix equation gives rise to $I_1, I_2, \cdots, I_N$ which are then used to determine the radiation pattern and other antenna parameters.

(a) Radiation pattern:

$$\bar{A} = \hat{z} \frac{\mu e^{-j\beta r}}{4\pi r} \int_0^L e^{j\beta \hat{r}\cdot\bar{r}} I(z') dz'$$

$$= \hat{z} \frac{\mu e^{-j\beta r}}{4\pi r} \sum_{n=1}^{N} I_n \int_{z_n-\Delta/2}^{z_n+\Delta/2} e^{j\beta \hat{r}\cdot\bar{r}} F_n(z') dz'$$

$$= \hat{z} \frac{\mu e^{-j\beta r}}{4\pi r} \sum_{n=1}^{N} I_n \Lambda_n(\hat{r})$$

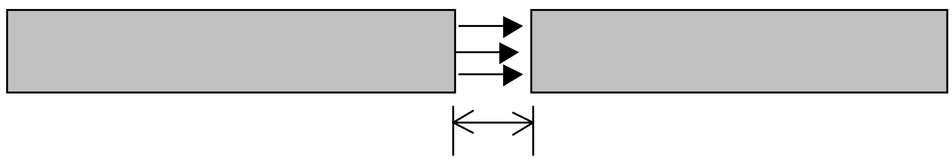
$$F_\theta(\theta) = \sin\theta \sum_{n=1}^{N} I_n \Lambda_n(\hat{r})$$

(b) Input impedance: If the matrix $\bar{\bar{Z}} = [f(z_m, z_n)]$ is inverted to be $\bar{\bar{Y}} = \bar{\bar{Z}}^{-1} = [y_{ij}]$, and the feeding point is at the center of k-th segment, then the input impedance $Z_A = y_{kk}^{-1}$.

- **Treatment of the excitation:**

1. Delta gap approximation:

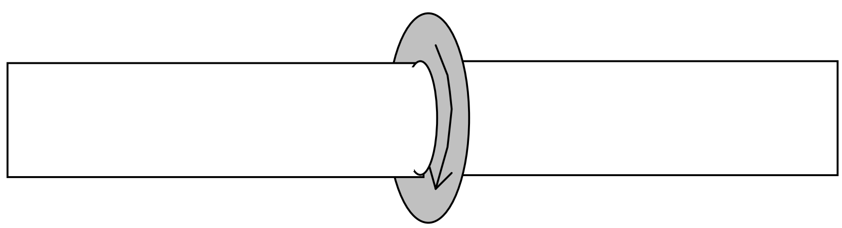
$$E_z^i(z_m) = \begin{cases} V_0/\delta, & \text{if segment m is feed point.} \\ 0, & \text{Elsewhere,} \end{cases}$$



2. Frill generator

$$E_z^i(z_m) = \frac{1}{2\ln(b/a)}\left(\frac{e^{-j\beta R_1}}{R_1} - \frac{e^{-j\beta R_2}}{R_2}\right),$$

$$R_1 = \sqrt{z_m^2 + a^2}, \quad R_2 = \sqrt{z_m^2 + b^2}$$



3. Plane wave incidence (for scattering application):

$E_z^i(z_m)$ is the incident wave at location z_m .

- **Galerkin's method:**

The testing function is the same as the basis function:

$$\sum_{n=1}^{N} I_n \int_{z_m-\Delta/2}^{z_m+\Delta/2} w_m(z) f(z, z_n) dz = -\int_{z_m-\Delta/2}^{z_m+\Delta/2} w_m(z) E_z^i(z)\, dz,$$

$$m = 1, 2, \cdots, N$$

The impedance matrix:

$$Z_{mn} = \int_{z_m-\Delta/2}^{z_m+\Delta/2} w_m(z) f(z, z_n) dz$$

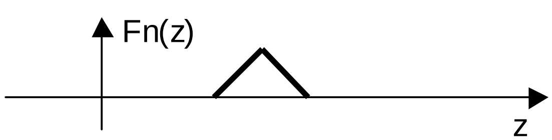
The excitation vector:

$$V_m = -\int_{z_m-\Delta/2}^{z_m+\Delta/2} w_m(z) E_z^i(z)\, dz,$$

The matrix equation:

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, \quad m = 1, 2, 3, \cdots, N$$

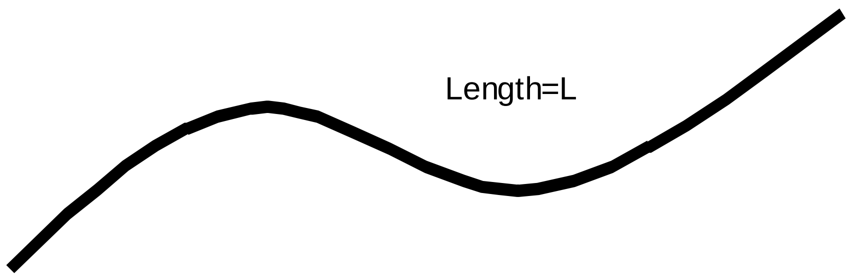
- **Linear basis functions (first order):**



$$F_n(z) = \begin{cases} 1 - |z - z_n| / \Delta, & \text{for } |z - z_n| \le \Delta \\ 0, & \text{Elsewhere} \end{cases}$$

Advantage: Higher accuracy for the same number of segments compared to the pulse basis function.

- **Wire antennas of arbitrary structure:**

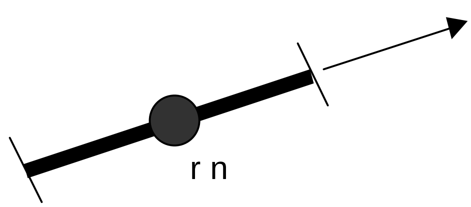


Integral equation:

$$\frac{\hat{t}}{j\omega\varepsilon}\cdot\int_0^L\left(\frac{\partial^2 g}{\partial l^2}+\beta^2 g\right)\hat{l}\,'I(l\,')dl\,'=-\hat{t}\cdot\overline{E}^i(l),\quad l\in[0,L]$$

Gridding: Divide the wire into segments. The grid points are denoted by 3D vectors as:

$$\overline{r}_1,\overline{r}_2,\overline{r}_3,\cdots,\overline{r}_N$$



On the n-th segment, the source vector $\overline{r}\,'$ is described by a local variable t :

$$\overline{r}\,'=\overline{r}_n+\hat{l}\,'\,t\,\Delta/2,\quad t\in[-1,1]$$

Basis function:

$$\overline{f}_n(r)=\begin{cases}\hat{l}\,'(1-|t|), & \overline{r}\in\text{Segment-n, t}\in[\text{-1,1}]\\ 0, & \text{Otherwise.}\end{cases}$$

Testing: Galerkin's method.