EE461g: Introduction to Electronic Circuits

Lecture 3: Diode as a Transfer Function & Half-Wave Rectifier

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Outline

Review: Transfer Function

Transfer function: Relationship between output and input

$$V_{out} = f(V_{in})$$

For linear circuits:

$$V_{out} = A \cdot V_{in} \quad \text{(constant gain)}$$

For diode circuits:

- Diodes introduce **nonlinearity**
- Output is not simply a scaled version of input
- We use piecewise linear models to approximate behavior

Recall: Three Piecewise Linear Diode Models

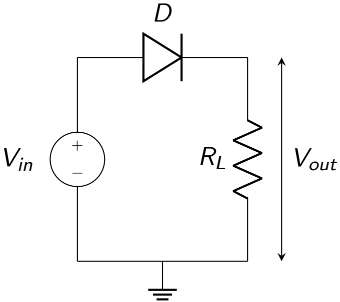
From Lecture 2, we have three models of increasing accuracy:

1. **Model 1 (Ideal):** $V_D = 0$ when ON
2. **Model 2 (Constant V_f):** $V_D = V_f$ when ON
3. **Model 3 ($V_f + r_d$):** $V_D = V_f + I_D \cdot r_d$ when ON

Today: Apply these models to analyze **rectifier** and **voltage limiter** circuits

Half-Wave Rectifier: Circuit Configuration

Goal: Allow current to flow through load in only one direction



Components:

- Voltage source V_{in}
- Diode D (in series)
- Load resistance R_L

Output: Voltage across R_L

$$V_{out} = V_{R_L}$$

DC Analysis First (Ideal Diode)

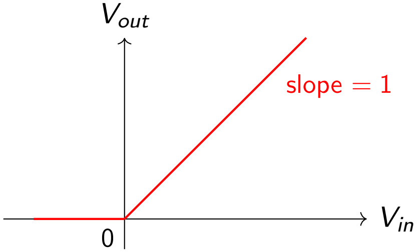
Apply a DC voltage V_{in} using **Model 1 (Ideal)**:

When $V_{in} > 0$**:**

- Diode turns ON
- Current flows through R_L
- $V_{out} = V_{in}$

When $V_{in} < 0$**:**

- Diode is OFF
- No current flows
- $V_{out} = 0$



DC Transfer Characteristic (Ideal)

Sinusoidal Input (Ideal Diode)

Now apply: $V_{in}(t) = V_m \sin(\omega t)$

Positive half-cycle ($V_{in} > 0$):

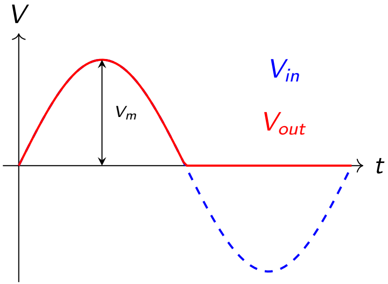
- Diode ON
- $V_{out} = V_{in}$

Negative half-cycle ($V_{in} < 0$):

- Diode OFF
- $V_{out} = 0$

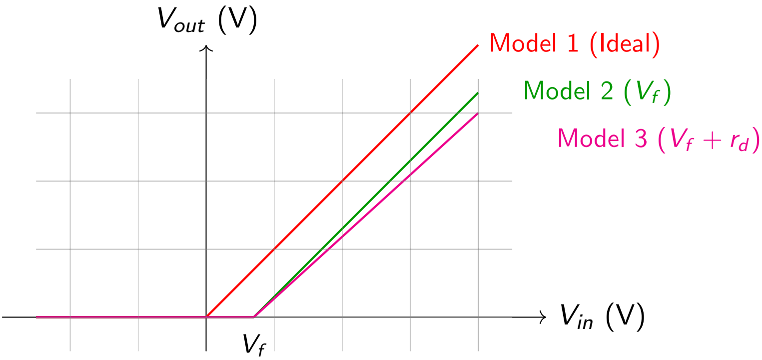
Result:

- Unidirectional current
- "Rectified" waveform



Output follows input exactly during positive half-cycle

Effect of Diode Model on Transfer Function

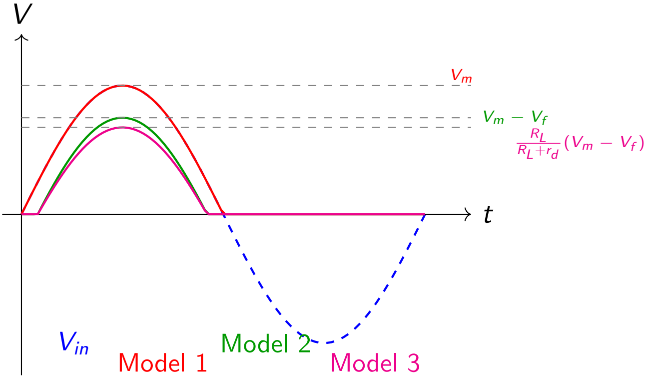


Key observations:

- Model 2 shifts turn-on to $V_{in} = V_f$ and reduces output by V_f
- Model 3 further reduces slope due to r_d voltage divider

Time-Domain Waveforms: Model Comparison

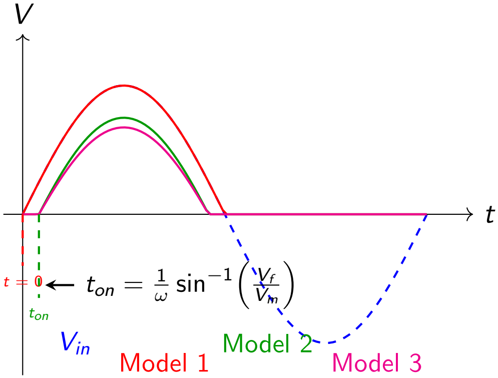
Input: $V_{in}(t) = V_m \sin(\omega t)$



Peak reduction: V_f drops peak, r_d further reduces it

Time-Domain Waveforms: Turn-On Delay

Input: $V_{in}(t) = V_m \sin(\omega t)$



Turn-on delay: Models 2 & 3 don't conduct until $V_{in} > V_f$

The Problem with r_d

Using Model 3: $V_D = V_f + I_D \cdot r_d$

The voltage across r_d:

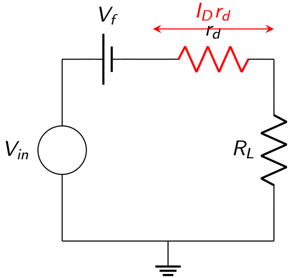
$$V_{r_d} = I_D \cdot r_d$$

This voltage is "stolen" from the load!

Power lost in diode:

$$P_{loss} = I_D^2 \cdot r_d$$

This energy was meant for the load!



Voltage Divider Effect

With diode ON:

$$V_{load} = V_{in} - V_f - I_D \cdot r_d$$

As current increases, more voltage is lost across r_d!

Efficiency:

$$\eta = \frac{P_{load}}{P_{source}} = \frac{I_D^2 R_L}{I_D(V_{in} - V_f)} = \frac{R_L}{R_L + r_d}$$

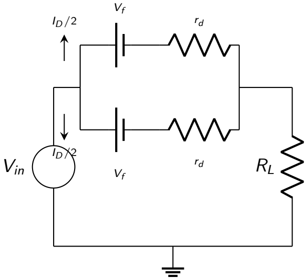
Example: With $R_L = 100\ \Omega$ and $r_d = 2.6\ \Omega$:

$$\eta = \frac{100}{100 + 2.6} = 97.5\%$$

We lose 2.5% of power in the diode's r_d!

Solution: Parallel Diodes

Place two diodes in parallel:



Current splits:

- Each diode carries $I_D/2$

V_f remains the same:

- Same turn-on voltage

r_d is halved:

$$r_{d,parallel} = \frac{r_d}{2}$$

Two resistors in parallel!

Parallel Diodes: Analysis

Single diode:

$$V_D = V_f + I_D \cdot r_d$$

Two parallel diodes:

$$V_D = V_f + I_D \cdot \frac{r_d}{2}$$

Result:

- Voltage loss across dynamic resistance is halved
- More power delivered to load
- Trade-off: cost and board space vs efficiency

Generalization: n diodes in parallel:

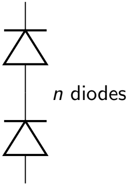
$$r_{d,eff} = \frac{r_d}{n}$$

Common in power electronics for high-current applications!

What About Series Diodes?

We tried parallel: V_f stays same, r_d is halved

What if we put diodes in series instead?



Two diodes in series:

$$V_{f,total} = 2V_f$$

$$r_{d,total} = 2r_d$$

n diodes in series:

$$V_{f,total} = n \cdot V_f$$

$$r_{d,total} = n \cdot r_d$$

This gives us a **controllable turn-on voltage**!

Application: Voltage limiting (clipping) circuits

Voltage Limiter: Motivation

Goal:

- Protect sensitive circuits from overvoltage
- Limit signal swing to a defined range

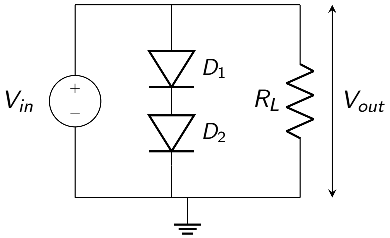
Idea: Use series diodes to set a clipping threshold

- When input exceeds threshold, diodes turn ON
- When ON, diodes "clamp" the output voltage

Clipping threshold:

$$V_{clip} = n \cdot V_f \quad (n = \text{number of series diodes})$$

Voltage Limiter: Circuit Configuration



Configuration:

- Diodes in series (D1, D2)
- In parallel with load R_L
- Diodes point toward ground

Clipping threshold:

$$V_{clip} = 2V_f \approx 1.4 \text{ V}$$

(for two Si diodes)

Voltage Limiter: Operating Principle

When $|V_{in}| < n \cdot V_f$**:**

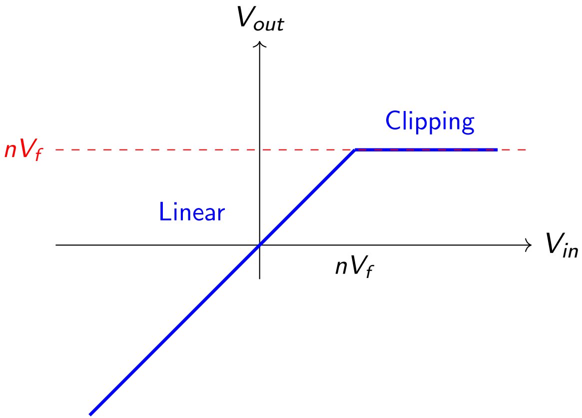
- Diodes are OFF (not enough voltage to turn on)
- $V_{out} = V_{in}$ (signal passes through)

When $V_{in} > n \cdot V_f$**:**

- Diodes turn ON
- V_{out} is clamped to $n \cdot V_f$
- Excess voltage dropped elsewhere

Result: Output is "clipped" at $V_{clip} = n \cdot V_f$

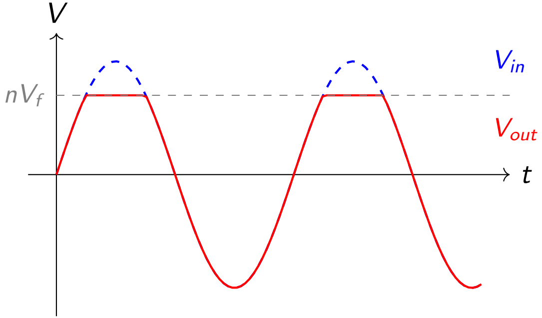
Voltage Limiter: Transfer Function



- **Linear region:** $V_{out} = V_{in}$ (diodes OFF)
- **Clipping region:** $V_{out} = n \cdot V_f$ (diodes ON)

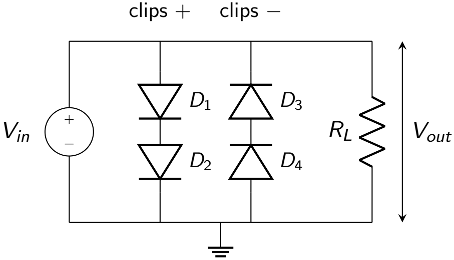
Voltage Limiter: Time-Domain Waveforms

Input: $V_{in}(t) = V_m \sin(\omega t)$ with $V_m > n \cdot V_f$



Flat-top clipping when input exceeds threshold

Bidirectional Limiter: Circuit

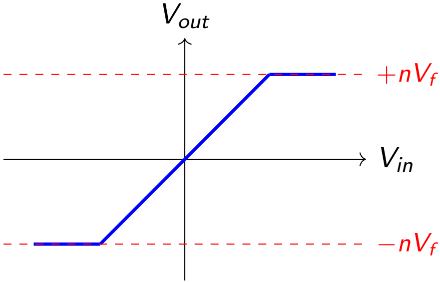


D1-D2: Clip positive voltages **D3-D4:** Clip negative voltages

Bidirectional Limiter: Transfer Function

Goal:

- Limit voltage swing in **both** directions
- Symmetric clipping for AC signals



Solution: Two sets of series diodes in parallel, opposite polarity

Bidirectional Limiter: Operating Principle

When $|V_{in}| < n \cdot V_f$**:**

- All diodes are OFF
- $V_{out} = V_{in}$

When $V_{in} > +n \cdot V_f$**:**

- D1-D2 turn ON (forward biased)
- V_{out} clamped to $+n \cdot V_f$

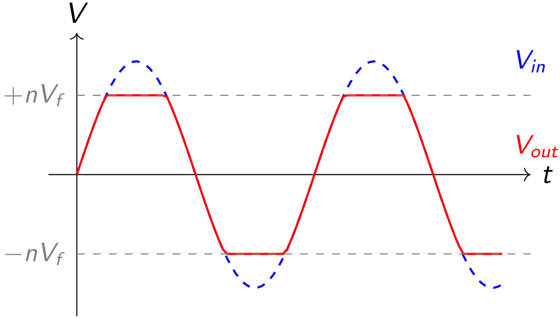
When $V_{in} < -n \cdot V_f$**:**

- D3-D4 turn ON (forward biased)
- V_{out} clamped to $-n \cdot V_f$

Result: Symmetric limiting around zero

Bidirectional Limiter: Time-Domain Waveforms

Input: $V_{in}(t) = V_m \sin(\omega t)$ with $V_m > n \cdot V_f$



Symmetric flat-top and flat-bottom clipping

Zener Diodes: Motivation

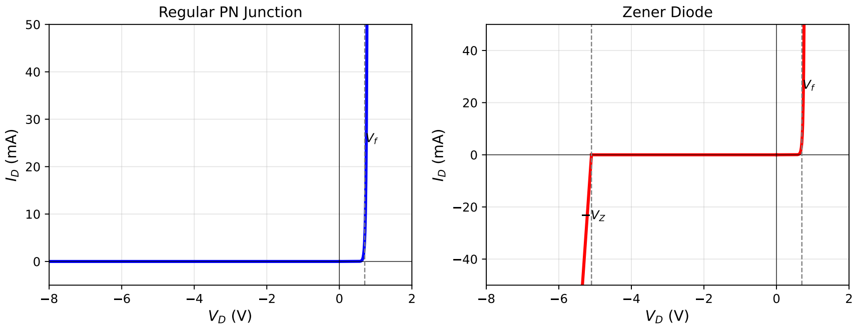
Problem: Multiple diodes in series/parallel is cumbersome!

- Need many diodes to reach desired clipping voltage
- Takes up board space
- Adds cost

Solution: Zener diodes

- Provide precise voltage limiting with a **single device**
- Replace n diodes with one Zener of chosen V_Z
- Available in standard voltages (3.3V, 5.1V, 12V, etc.)

Zener Diode: I-V Characteristic



Key difference: Zener conducts in **reverse** at breakdown voltage V_Z

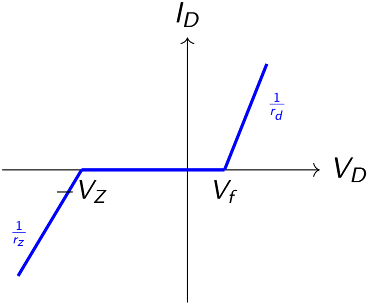
Zener Diode: Piecewise Linear Model

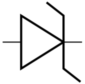
Three operating regions:

1. **Forward bias** ($V_D > V_f$): ON
2. **Off region** ($-V_Z < V_D < V_f$): OFF
3. **Reverse breakdown** ($V_D < -V_Z$): ON

With dynamic resistance:

- Forward: slope $= 1/r_d$
- Reverse: slope $= 1/r_z$





Zener symbol

Zener Diode: Model Parameters

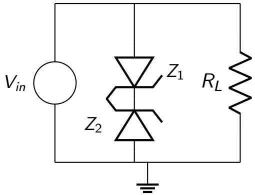
Key parameters:

- V_f: Forward voltage drop (≈ 0.7 V for Si)
- V_Z (or V_{BK}): Zener breakdown voltage
  - Designed value (e.g., 3.3V, 5.1V, 12V)
  - Sets the clipping threshold in reverse
- r_z: Dynamic resistance in breakdown region
  - Optional refinement for accurate analysis
  - Typically small (few ohms)

Advantage: Single device replaces multiple series diodes!

Simplified Bidirectional Limiter with Zeners

Replace multiple diodes with Zeners:



Back-to-back Zeners

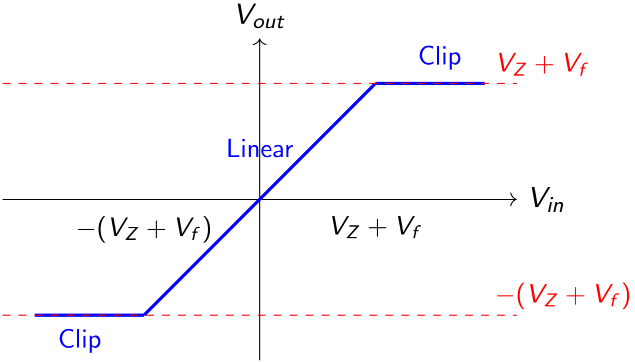
Clipping levels:

- Positive: $V_Z + V_f$
- Negative: $-(V_Z + V_f)$

Example: With $V_Z = 5.1$ V:

- Clips at ± 5.8 V
- Replaces ~ 8 series diodes!

Zener Limiter: Transfer Function



Symmetric clipping at $\pm(V_Z + V_f)$ with a single pair of Zeners!

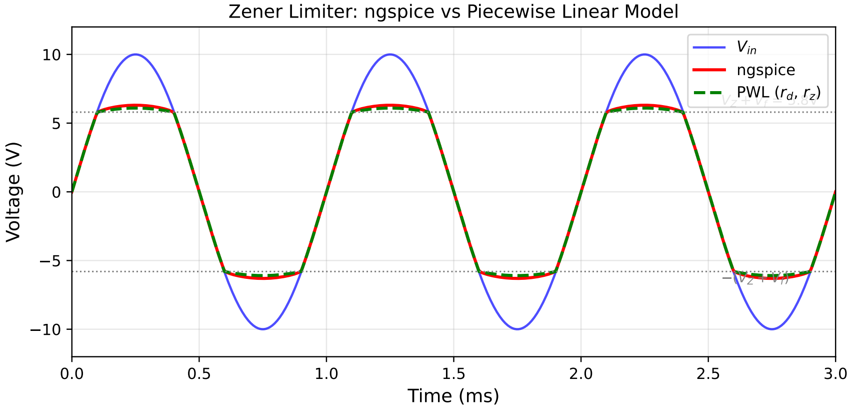
SPICE Simulation

Verify our analysis with SPICE:

1. Half-wave rectifier with realistic diode model
2. Single-sided voltage limiter
3. Bidirectional voltage limiter
4. Zener-based limiter

Compare: SPICE results vs piecewise linear predictions

Zener Limiter: Transient Response



- **SPICE model**: Exponential diode characteristics with $V_Z = 5.1$ V
- **PWL model**: Includes r_d and r_z for gradual clipping slope
- Both show symmetric clipping at $\pm(V_Z + V_f) \approx \pm 5.8$ V

PWL Diode Models in SPICE: A Cautionary Tale

Goal: Implement piecewise linear diode models in SPICE

The problem:

- SPICE requires complete circuits with defined node voltages
- An "open circuit" (infinite resistance) leaves nodes floating
- Floating nodes cause convergence failures or incorrect results

Test circuit: Bidirectional voltage limiter (same as earlier)

- Three diodes in series for positive and negative limiting
- We'll compare real SPICE diodes vs. PWL implementations

Reference: Real SPICE Diode Model

1_real_diode.png

PWL Attempt 1: Ideal Diode (Open When OFF)

2_ideal_diode.png

PWL Attempt 2: Ideal Diode + V_f (Open When OFF)

3_ideal_vf.png

PWL Attempt 3: $V_f + R_s$ (Open When OFF)

4_vf_rs_open.png

PWL Solution: Add R_{off} for DC Path

5_vf_rs_roff.png

PWL Models in SPICE: Key Takeaways

When implementing PWL diode models in SPICE:

1. **Never use true open circuits**
   - Floating nodes cause incorrect results
   - SPICE needs a path to determine every node voltage
2. **Model the OFF state with large R_{off}**
   - Use $R_{off} \approx 1\ \mathrm{M\Omega}$ or larger
   - Small enough to avoid numerical issues
   - Large enough to approximate open circuit
3. **Complete PWL diode model:**
   - ON: V_f in series with R_s (or r_d)
   - OFF: Large R_{off} provides DC path

Rule of thumb: If a node can become isolated, add a high-value resistor to ground.

Summary

1. **Diodes create nonlinear transfer functions**
   - Output is not simply a scaled version of input
2. **Half-wave rectifier**
   - Allows current flow in only one direction
   - Parallel diodes reduce r_d losses
3. **Series diodes** increase V_f and r_d
   - Useful for voltage limiting (clipping)
4. **Voltage limiters**
   - Single-sided or bidirectional clipping
   - Clipping threshold: $n \cdot V_f$
5. **Zener diodes**
   - Simplify voltage limiting with built-in V_Z
   - Single device replaces multiple series diodes