EE461g: Introduction to Electronic Circuits

Lecture 4: Half-Wave Rectifier with Capacitor Filter

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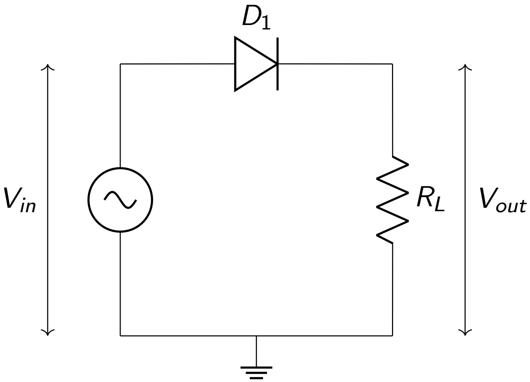
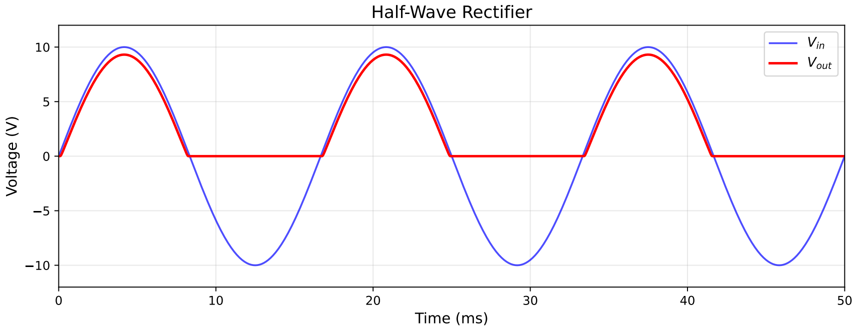
Outline

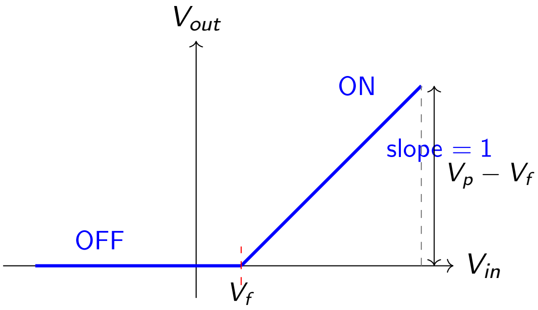
Half-Wave Rectifier Circuit



Half-Wave Rectifier: Waveforms

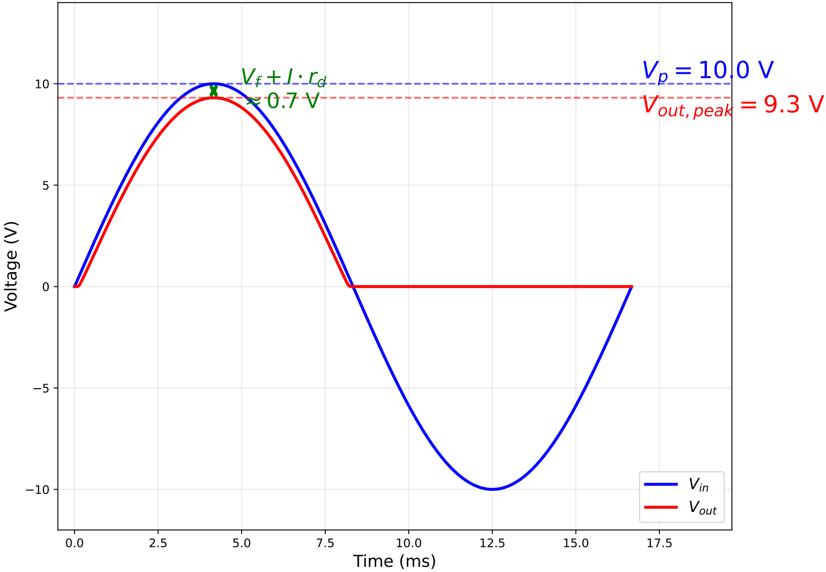


Half-Wave Rectifier: Transfer Function

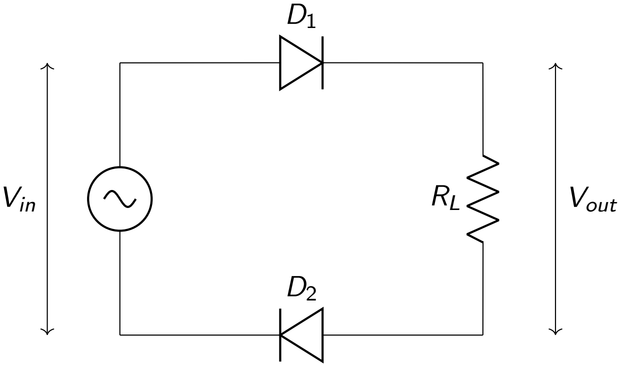


$$V_{out} = \max(0,\ V_{in} - V_f)$$

Peak Voltage Analysis

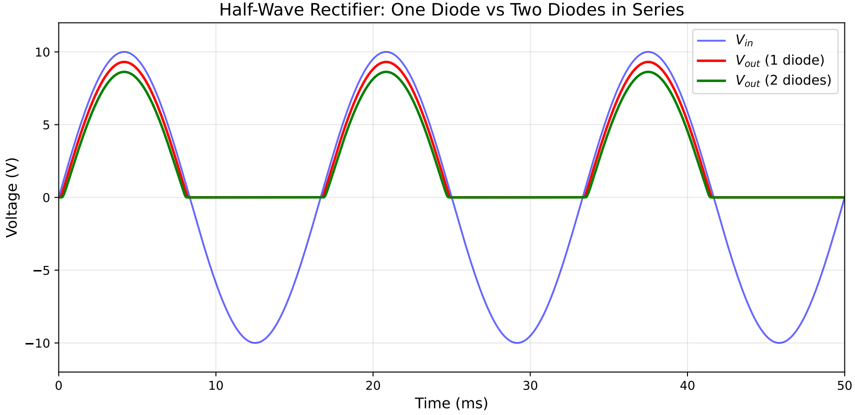


Half-Wave Rectifier: Two Diodes in Series

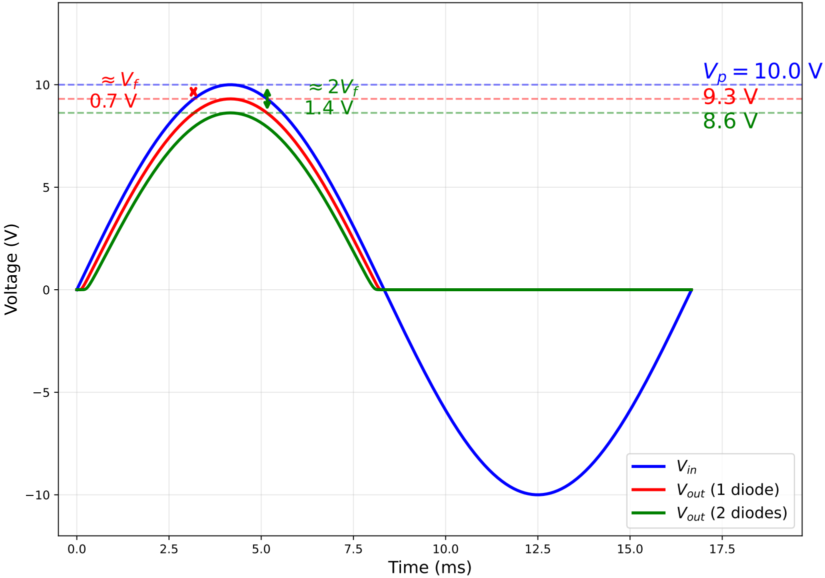


Both diodes conduct during positive half-cycle → voltage drop is $2V_f$

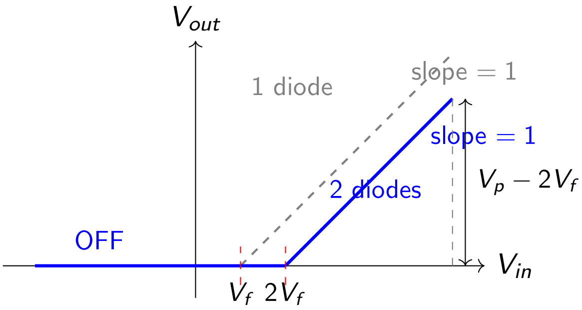
Two Diodes: Waveform Comparison



Two Diodes: Peak Voltage Comparison

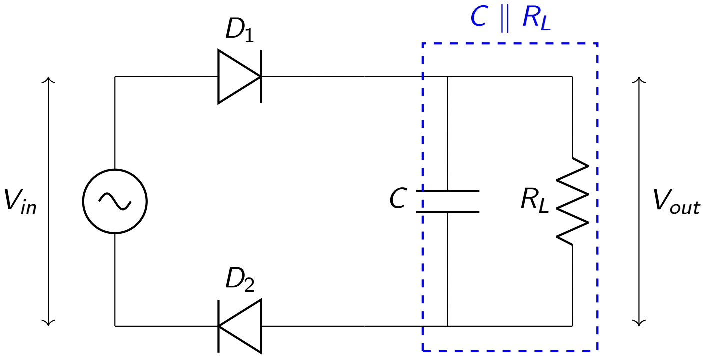


Two Diodes in Series: Transfer Function



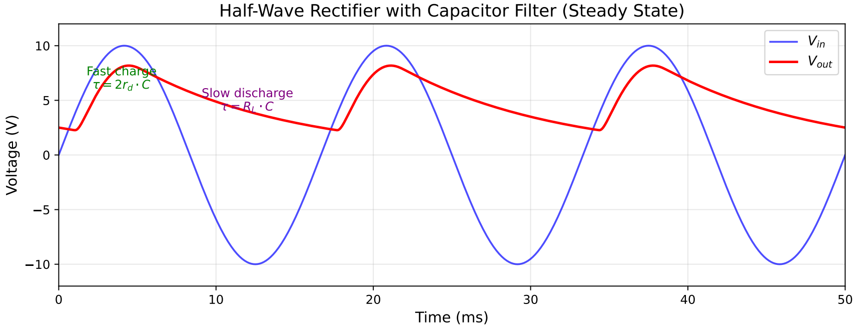
$$V_{out} = \max(0,\ V_{in} - 2V_f)$$

Adding a Filter Capacitor



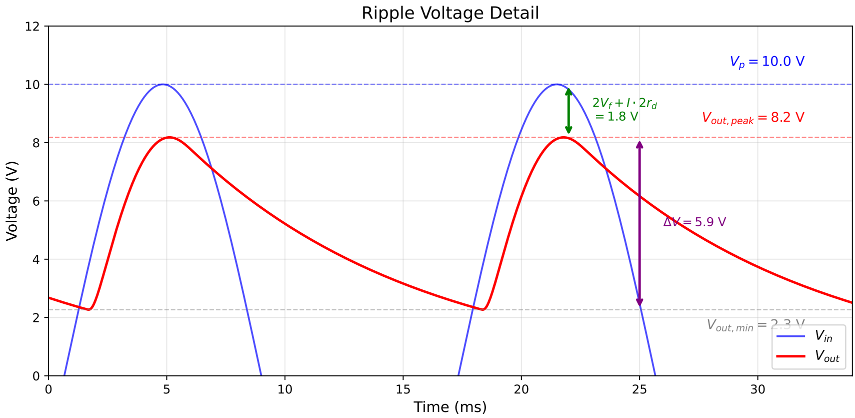
- Capacitor C and load R_L are in **parallel**
- Two diodes in series: voltage drop is $2V_f$

With Capacitor: Steady-State Waveforms



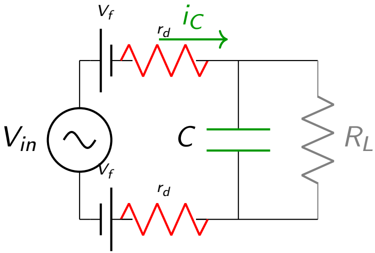
- **Fast charge** ($\tau = 2r_d \cdot C$): Capacitor charges quickly through diodes
- **Slow discharge** ($\tau = R_L \cdot C$): Capacitor discharges slowly through load

Ripple Voltage Detail



- Peak drop due to $2V_f + I \cdot 2r_d$ visible at output peak
- Ripple voltage ΔV: difference between max and min output

Diodes Forward Biased: Charging Phase



Condition: $V_{in} > V_{out} + 2V_f$

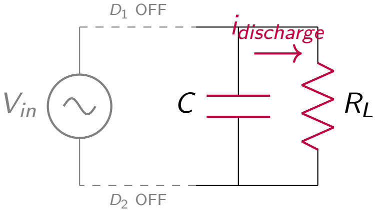
What happens:

- Both diodes turn ON
- Current flows through $2r_d$
- Capacitor charges quickly

Time constant:

$$\tau_{charge} = 2r_d \cdot C$$

Very small! ($2r_d = 2\Omega$, $C = 100\mu\text{F}$)



Condition: $V_{in} < V_{out}$

What happens:

- Both diodes turn OFF
- Capacitor isolated from source
- Discharges through R_L

Time constant:

$$\tau_{discharge} = R_L \cdot C$$

Much larger! ($R_L = 100\Omega$, $C = 100\mu\text{F}$)

Key Insight: Asymmetric Time Constants

Charging

Diodes ON

$\tau_{charge} = 2r_d \cdot C$

$= 2(1\Omega)(100\mu\text{F}) = 200\mu\text{s}$

→

Discharging

Diodes OFF

$\tau_{discharge} = R_L \cdot C$

$= (100\Omega)(100\mu\text{F}) = 10\text{ms}$

$$\tau_{charge} \ll \tau_{discharge}$$

because $2r_d \ll R_L$

Result: Capacitor charges quickly, discharges slowly → **smoothed DC output**

Ripple Voltage: Derivation

During discharge (diode OFF):

$$V_{out}(t) = V_p \cdot e^{-t/(R_L C)}$$

Approximation for small ripple ($\Delta V \ll V_p$):

- Discharge time $\approx T$ (one period)
- Use linear approximation: $e^{-x} \approx 1 - x$ for small x

Result:

$$\boxed{\Delta V \approx \frac{V_p}{f \cdot R_L \cdot C} = \frac{V_p \cdot T}{R_L \cdot C}}$$

where f is the input frequency and $T = 1/f$ is the period.

Ref: Razavi, Fundamentals of Microelectronics, 2nd ed., Sec. 3.5.1, Eq. 3.80

Design Equation for Capacitor

Given: Desired ripple ΔV, load R_L, peak voltage V_p, frequency f

Required capacitance:

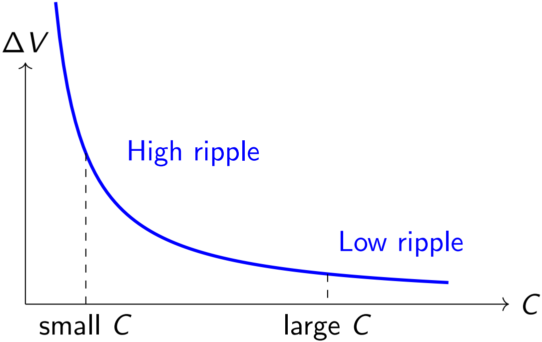
$$\boxed{C = \frac{V_p}{f \cdot R_L \cdot \Delta V}}$$

Example:

- $V_p = 10$ V, $f = 60$ Hz, $R_L = 1$ kΩ
- Desired $\Delta V = 1$ V (10% ripple)
- $C = \frac{10}{60 \times 1000 \times 1} = 167\ \mu$F

Ref: Razavi, Fundamentals of Microelectronics, 2nd ed., Sec. 3.5.1, Eq. 3.80

Effect of Capacitor Size

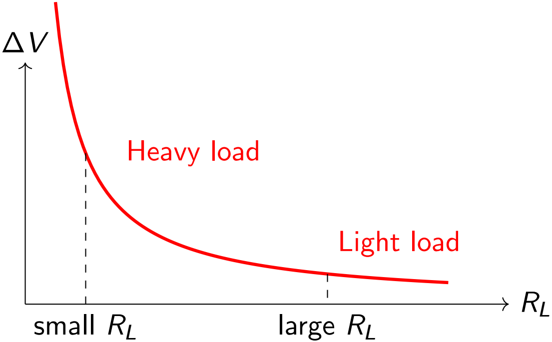


Larger capacitor:

- Longer discharge time constant $\tau = R_L \cdot C$
- Smaller ripple voltage
- Trade-off: cost, physical size, inrush current at startup

Ref: Razavi, Fundamentals of Microelectronics, 2nd ed., Sec. 3.5.1, Eq. 3.80

Effect of Load Resistance

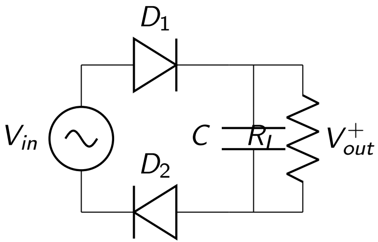


Larger load resistance (lighter load):

- Less current draw from capacitor
- Longer discharge time constant
- Smaller ripple voltage

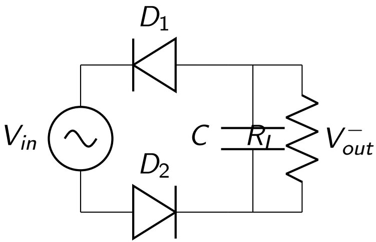
Ref: Razavi, Fundamentals of Microelectronics, 2nd ed., Sec. 3.5.1, Eq. 3.80

Original circuit:



Conducts on **positive** peaks

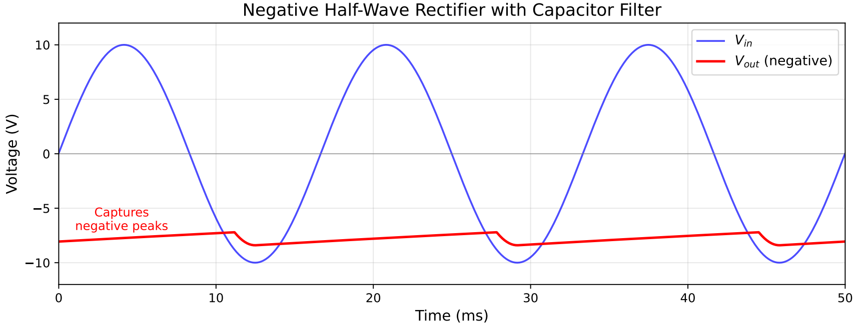
Reversed polarity:



Conducts on **negative** peaks

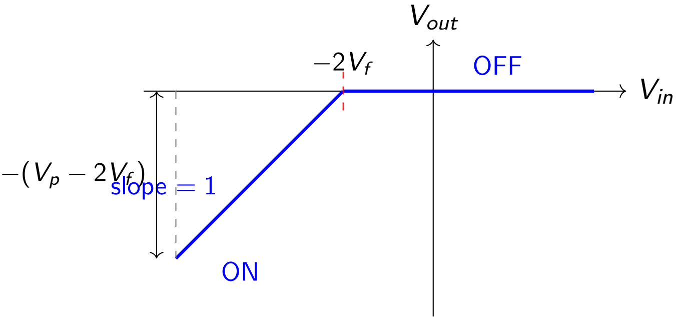
Key insight: Reversing both diodes captures the *valleys* instead of the peaks!

Negative Half-Wave Rectifier: Waveforms



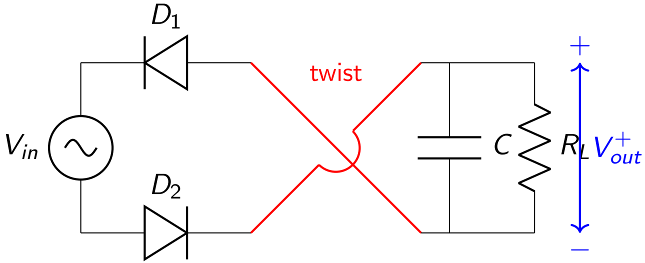
Output follows **negative** peaks of input — capacitor charges to negative peak voltage

Negative Half-Wave Rectifier: Transfer Function



$V_{out} = \min(0,\ V_{in} + 2V_f)$ — output is **negative**

Flipping the Output Reference

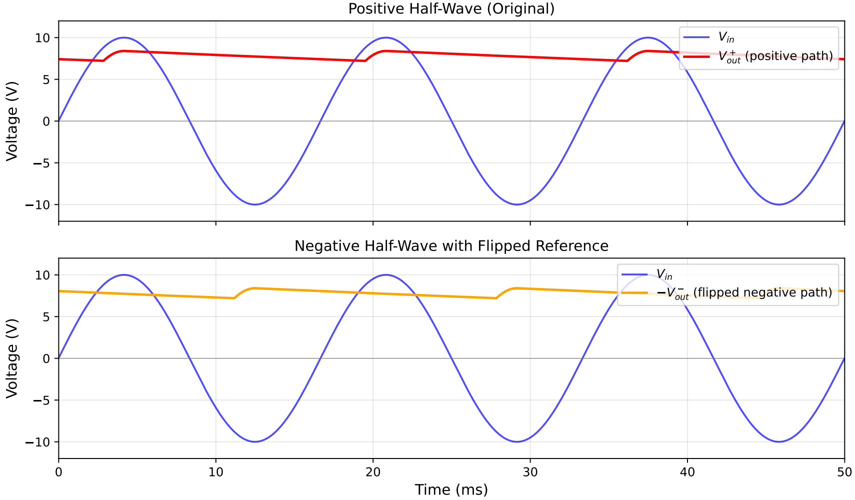


Key idea: Twist the connections so that:

- D_1 (top) → connects to **bottom** of R_L (negative terminal)
- D_2 (bottom) → connects to **top** of R_L (positive terminal)

Result: Same circuit, same waveform — but now **positive** output!

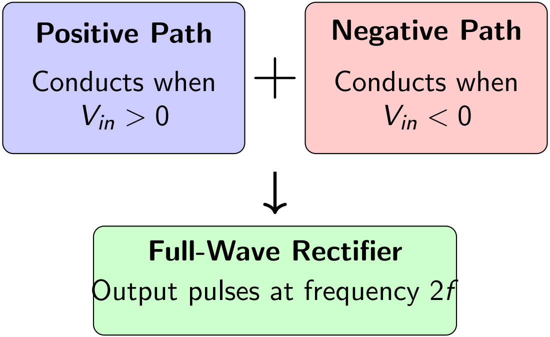
Two Paths to Positive Output



Observation: We can get positive DC from *either* half of the AC cycle!

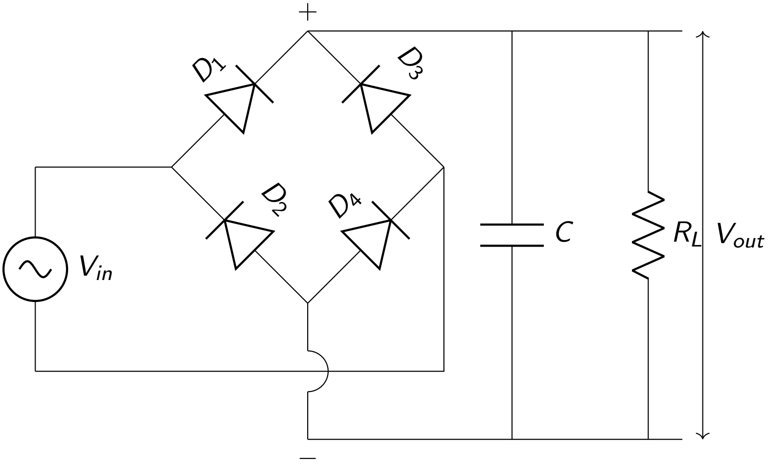
Combining Both Half-Waves

Idea: Use *both* halves of the input simultaneously!

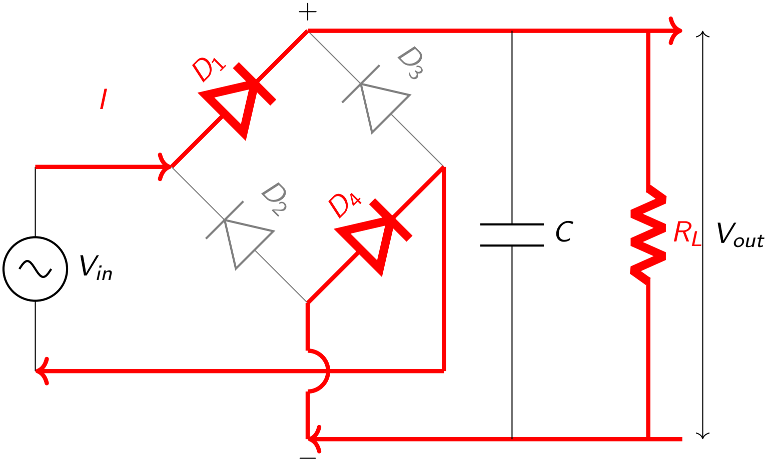


This is the **full-wave bridge rectifier**!

Full-Wave Bridge Rectifier Circuit

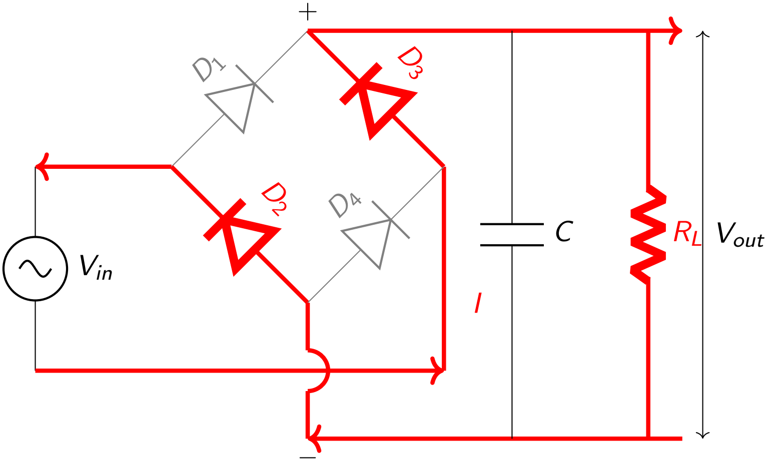


Positive Half-Cycle: D_1 and D_4 Conduct



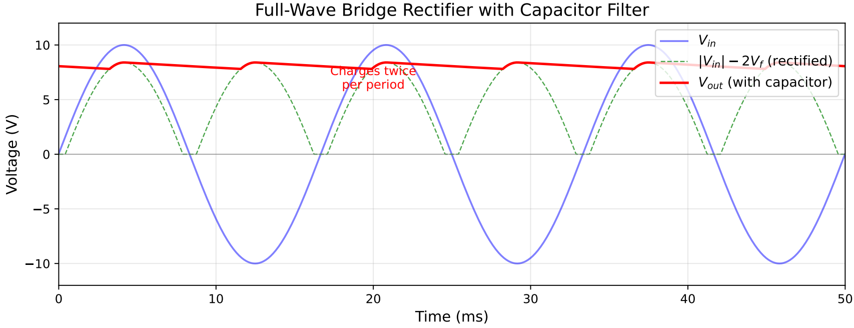
When $V_{in} > 0$: Current flows through D_1 and D_4

Negative Half-Cycle: D_3 and D_2 Conduct



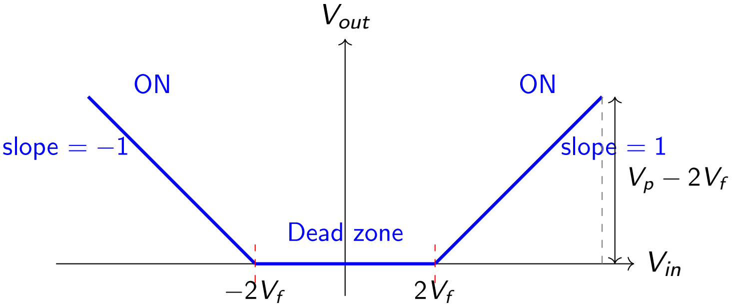
When $V_{in} < 0$: Current flows through D_3 and D_2

Full-Wave Rectifier: Waveforms



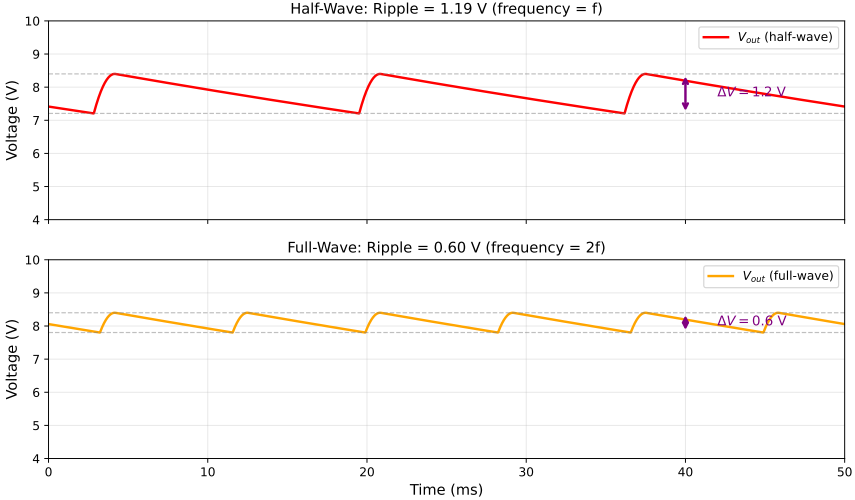
Key observation: Capacitor charges **twice per period** $\rightarrow$ ripple frequency is $2f$

Full-Wave Bridge Rectifier: Transfer Function



$V_{out} = \max(0, |V_{in}| - 2V_f)$ — output is always **positive**

Half-Wave vs Full-Wave Ripple



Same capacitor, same load — but **half the ripple** with full-wave!

Full-Wave Ripple Voltage

Half-wave: Discharge time $\approx T = 1/f$

$$\Delta V_{half} \approx \frac{V_p}{f \cdot R_L \cdot C}$$

Full-wave: Discharge time $\approx T/2 = 1/(2f)$

$$\boxed{\Delta V_{full} \approx \frac{V_p}{2f \cdot R_L \cdot C} = \frac{\Delta V_{half}}{2}}$$

Interpretation:

- Effective ripple frequency: $2f$ (twice the input frequency)
- Capacitor has less time to discharge before next charging pulse
- Same capacitor gives half the ripple!

Ref: Razavi, Fundamentals of Microelectronics, 2nd ed., Sec. 3.5.1, Eq. 3.94

Design Comparison

For same ripple specification:

Parameter	Half-Wave	Full-Wave
Ripple frequency	f	$2f$
Required capacitance	C	$C/2$
Diode voltage drop	$2V_f$	$2V_f$
Number of diodes	2	4

Trade-off:

- Full-wave uses 2 more diodes
- But requires half the capacitance for same ripple
- Generally preferred for power supplies

FULL BRIDGE RECTIFIER!!!

`https://www.electroboom.com/?p=544`

(or search YouTube for "ElectroBoom Full Bridge Rectifier")

Why watch this video?

- See a real full-wave bridge rectifier being built
- Entertaining and memorable explanation of how it works
- Shows actual oscilloscope waveforms of the output
- Demonstrates the effect of adding filter capacitors and inductors
- You'll never forget what a full bridge rectifier is!

Summary

1. **Half-wave rectifier with capacitor filter**
   - Asymmetric time constants: $\tau_{charge} = 2r_d C \ll \tau_{discharge} = R_L C$
   - Ripple: $\Delta V_{half} \approx V_p/(f \cdot R_L \cdot C)$
2. **Reversing diode polarity**
   - Captures negative peaks instead of positive
   - Flipping output reference gives same positive waveform
3. **Full-wave bridge rectifier**
   - Uses both half-cycles of input (4 diodes)
   - Capacitor charges twice per period
4. **Full-wave ripple**
   - Effective frequency: $2f \rightarrow \Delta V_{full} = \Delta V_{half}/2$
   - Half the ripple for the same capacitor!