EE461g: Introduction to Electronic Circuits

Lecture 9: NMOS MOSFET

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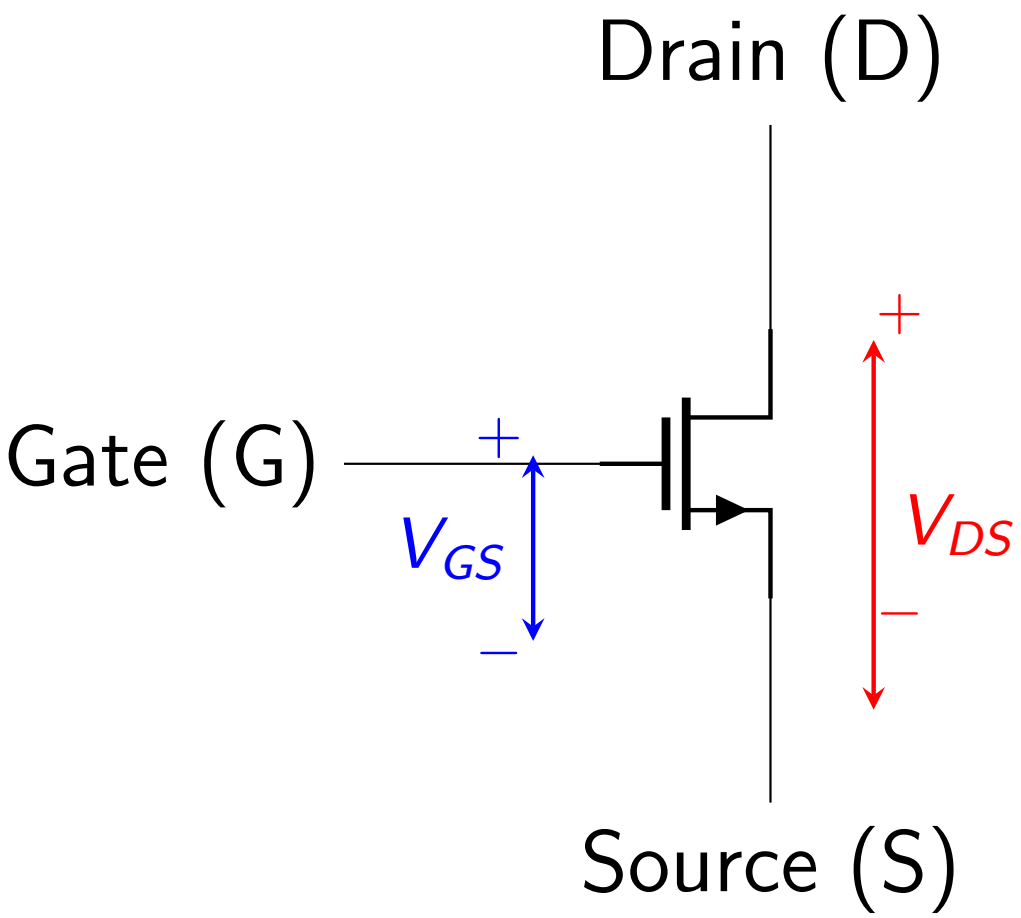
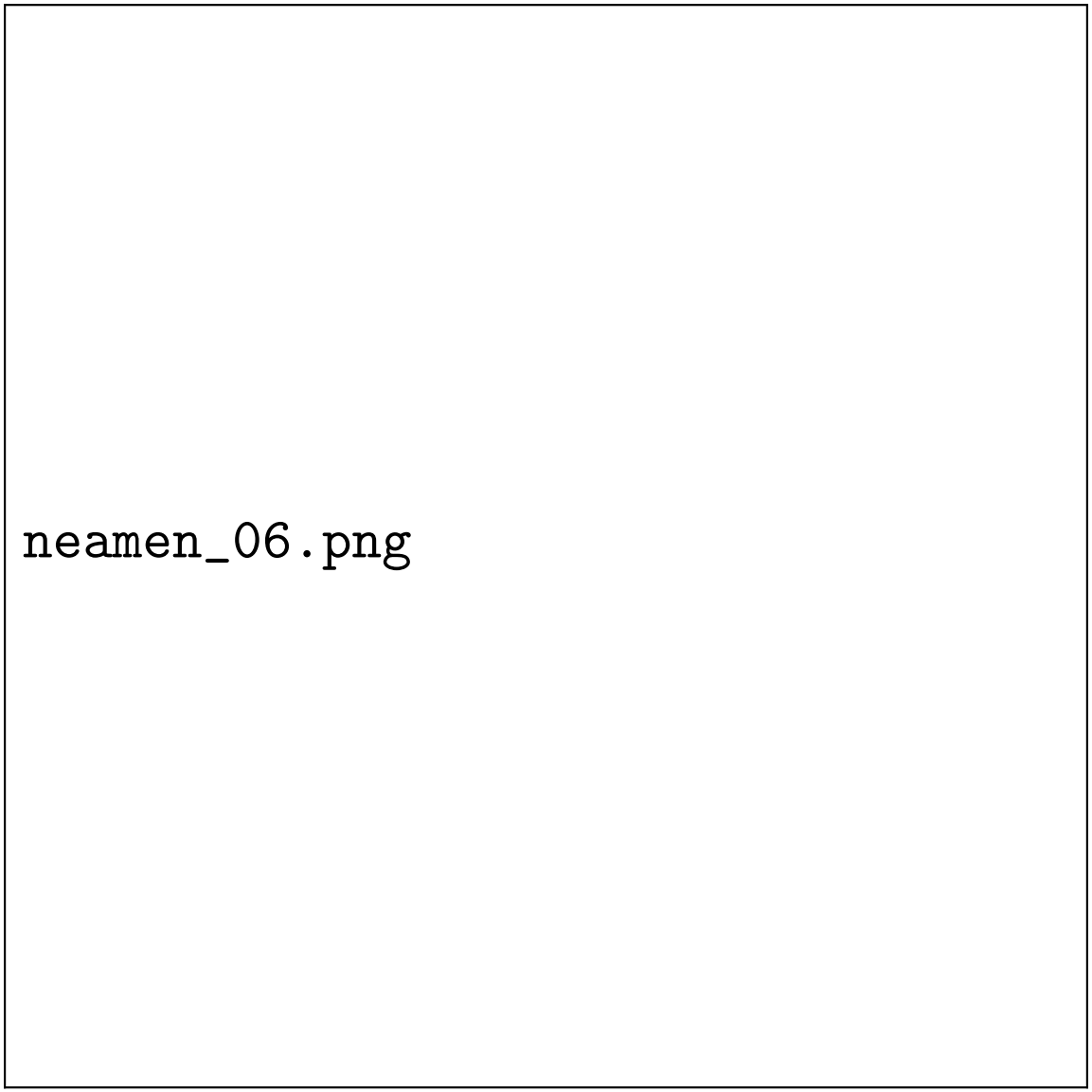
Outline

(Sec. 6.1) NMOS MOSFET Symbol



- **Gate (G):** Controls the channel (high impedance input)
- **Drain (D):** Current flows out of drain
- **Source (S):** Current flows into source (reference for V_{GS})



Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.

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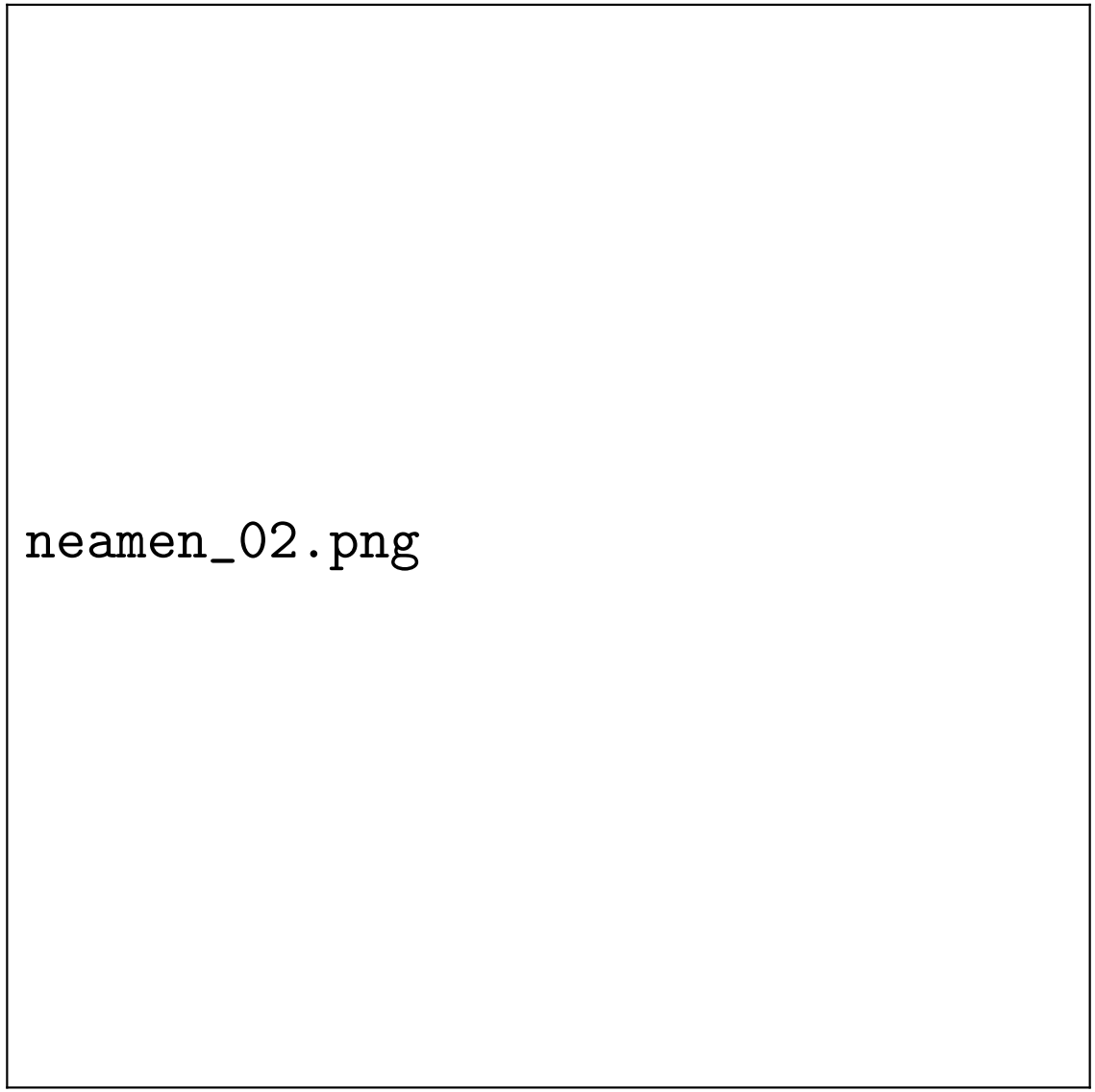
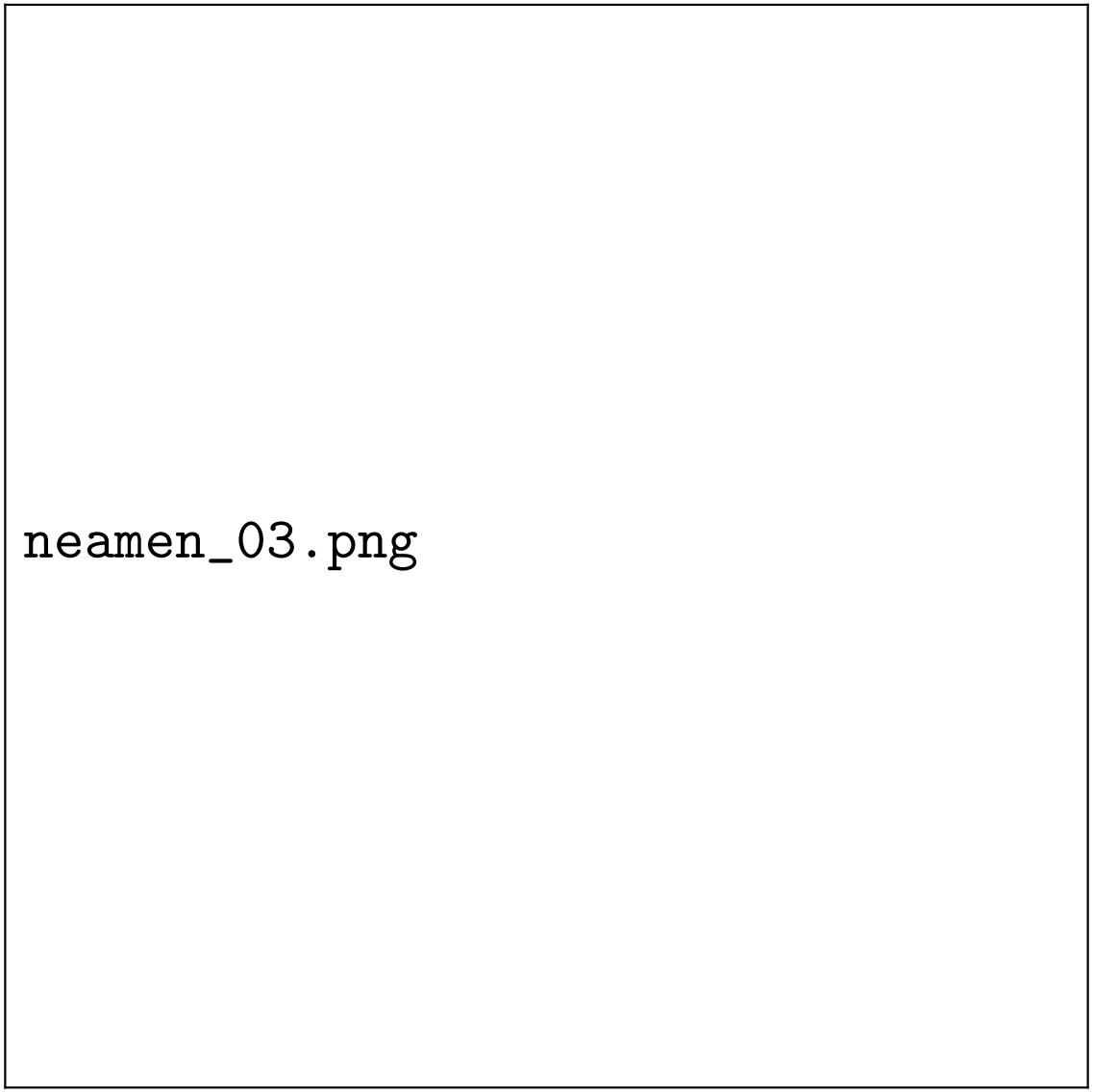
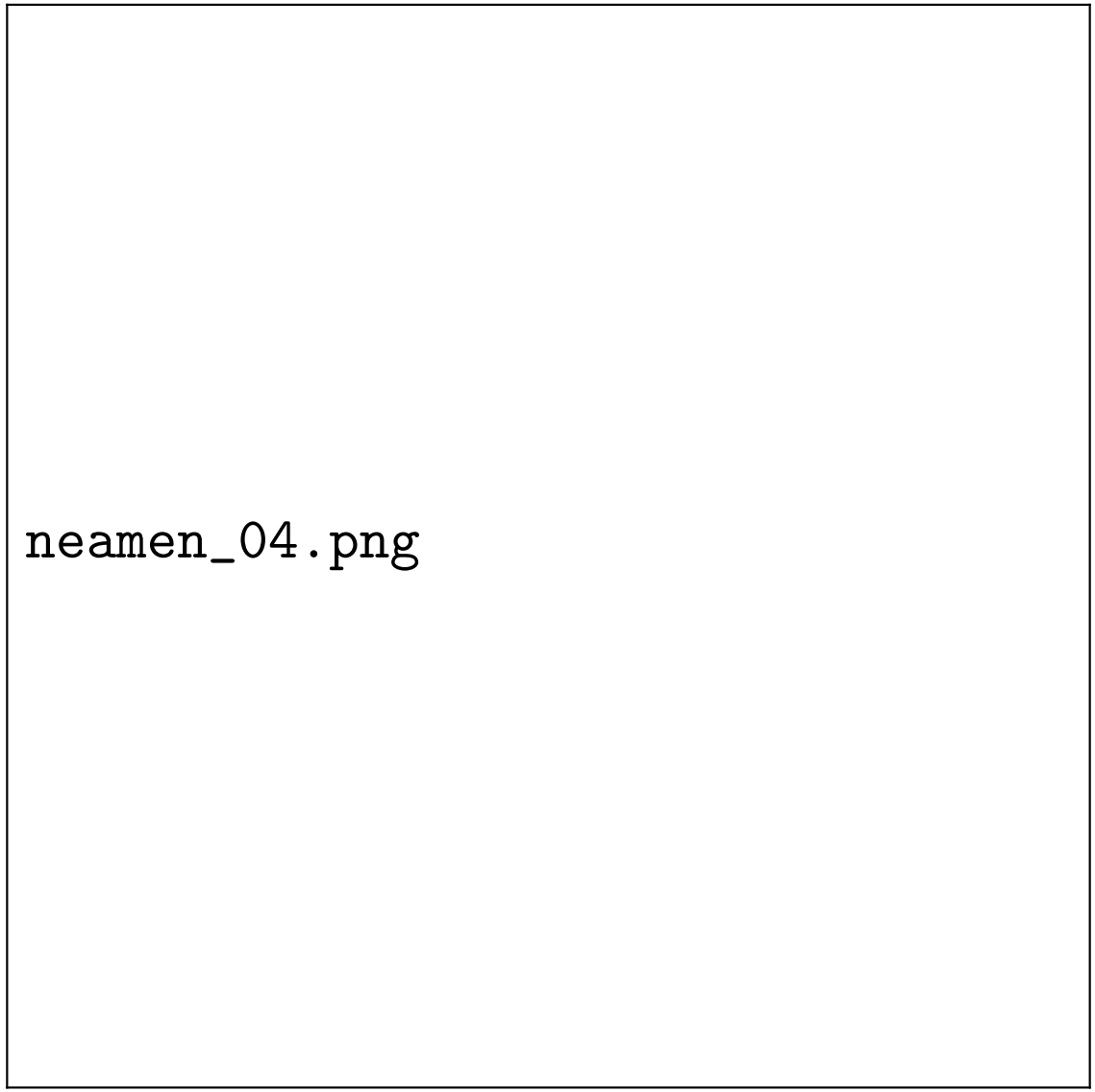
Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.



Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.



Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.



Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.

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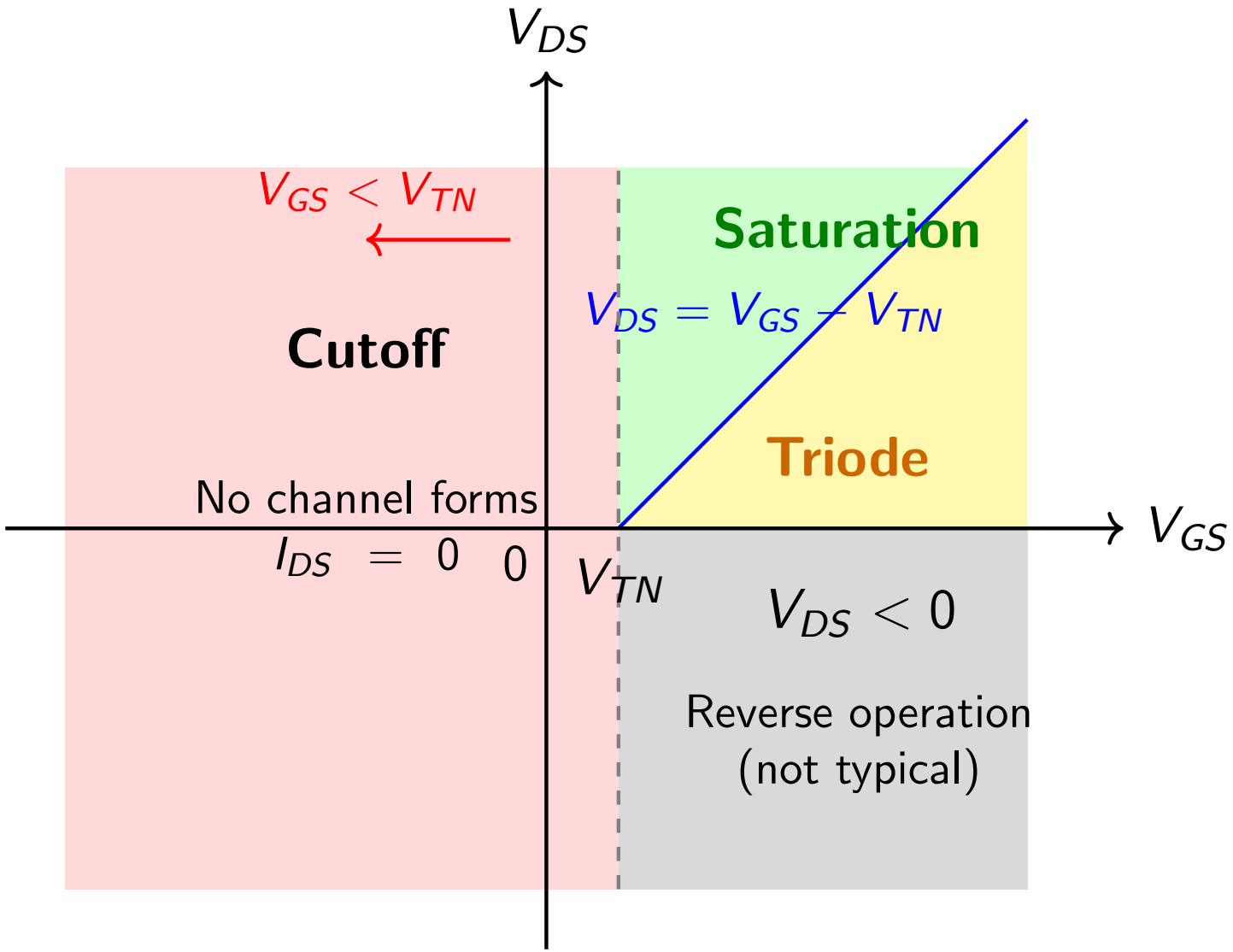
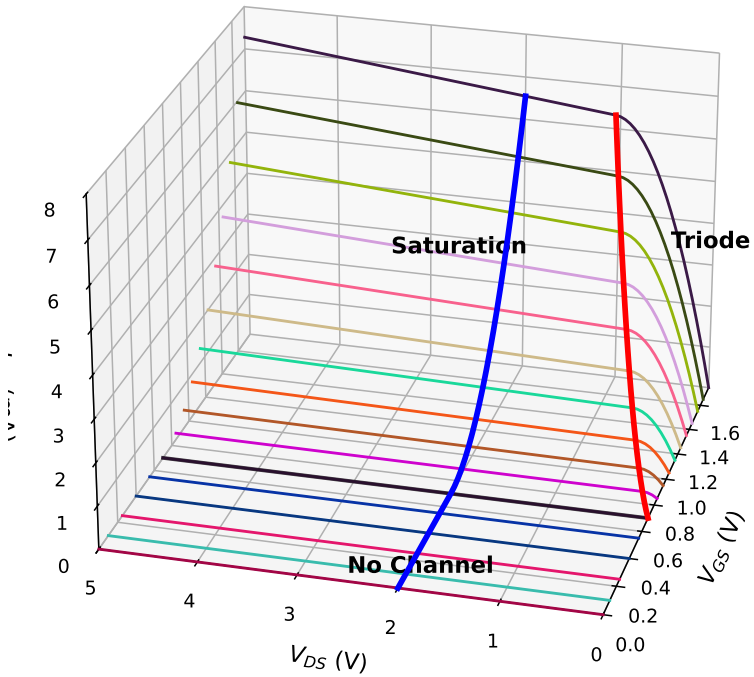
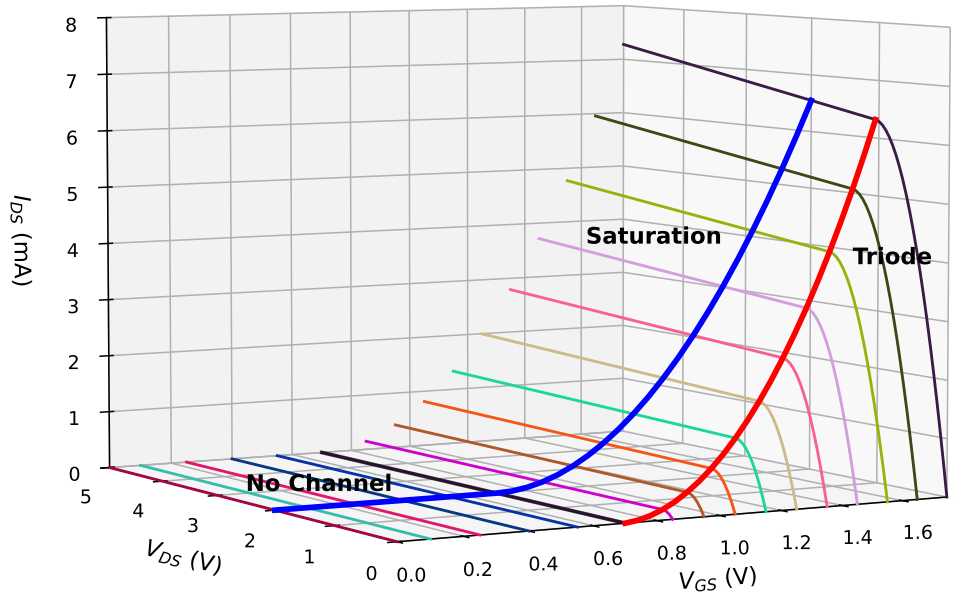
Source: Neamen, *Microelectronics: Circuit Analysis and Design*, 4th ed.

(Sec. 6.2) MOSFET Operating Space: V_{GS} vs V_{DS}

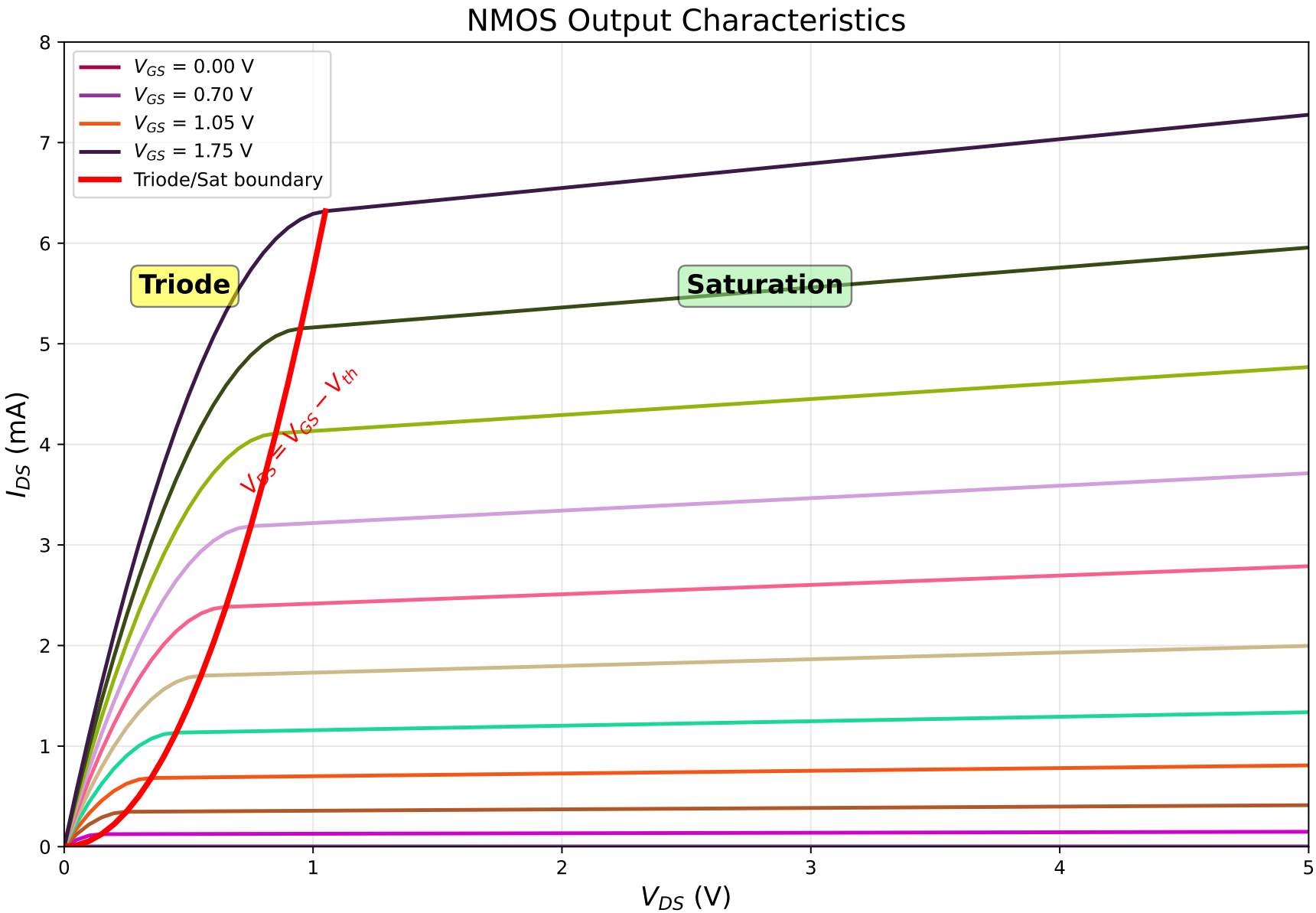


(Sec. 6.2.2) NMOS Output Characteristics – 3D View

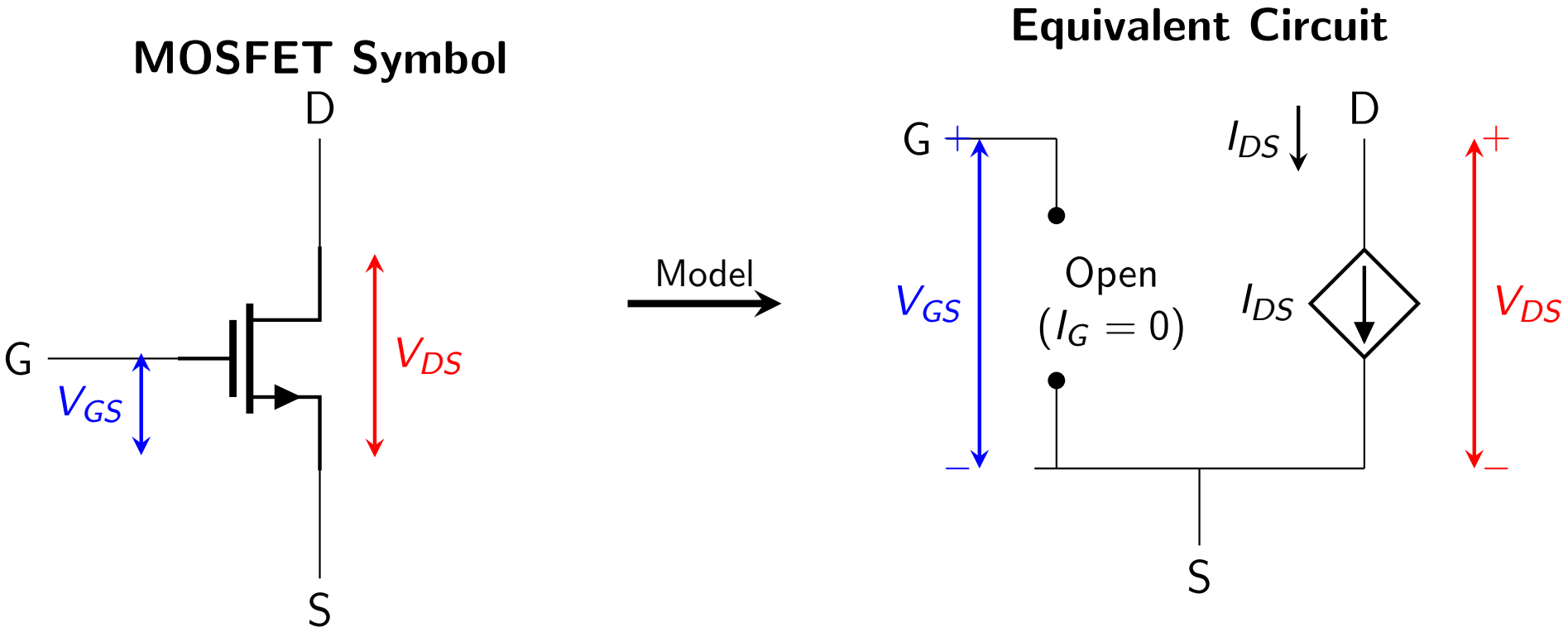




(Sec. 6.2.2) NMOS Output Characteristics – 2D View



(Sec. 6.3.1) NMOS Large-Signal Model (Square Law)



$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2 \quad \text{(saturation)}$$

(Sec. 6.2.2) NMOS Operating Regions

Region	Condition	Current
Cutoff	$V_{GS} < V_{TN}$	$I_{DS} = 0$
Triode	$V_{GS} \geq V_{TN}$ $V_{DS} < V_{GS} - V_{TN}$	$I_{DS} = \frac{K_n}{2}[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$
Saturation	$V_{GS} \geq V_{TN}$ $V_{DS} \geq V_{GS} - V_{TN}$	$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$

$$K_n = \mu_n C_{ox} \frac{W}{L} \qquad \text{(process \& geometry dependent)}$$

- The quantity $(V_{GS} - V_{TN})$ is called the **overdrive voltage**
- Saturation boundary: $V_{DS} = V_{GS} - V_{TN}$

(Sec. 6.2.2) Deriving the Saturation Equation from Triode

Triode Region Equation:

$$I_{DS} = \frac{K_n}{2}\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$$

At the saturation boundary: $V_{DS} = V_{GS} - V_{TN}$

Substitute $V_{DS} = V_{GS} - V_{TN}$ into the triode equation:

$$\begin{aligned} I_{DS} &= \frac{K_n}{2}\left[2(V_{GS} - V_{TN}) \cdot (V_{GS} - V_{TN}) - (V_{GS} - V_{TN})^2\right] \\ &= \frac{K_n}{2}\left[2(V_{GS} - V_{TN})^2 - (V_{GS} - V_{TN})^2\right] \\ &= \frac{K_n}{2}(V_{GS} - V_{TN})^2 \end{aligned}$$

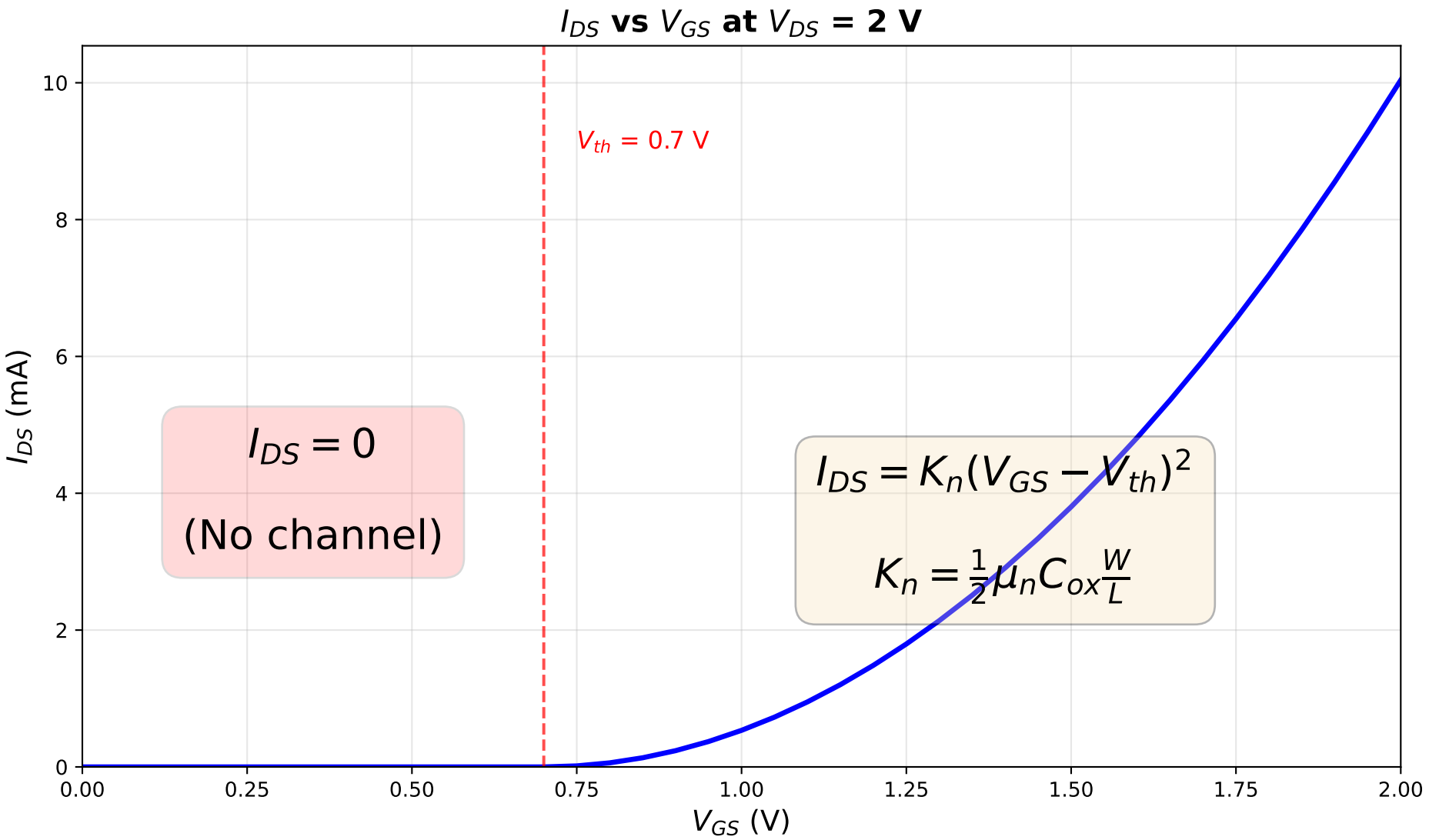
Saturation Region Equation:

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

(Sec. 6.2.2) Triode as a Parabola, Saturation as the Peak

triode_parabola.pdf

(Sec. 6.2.2) I_{DS} vs V_{GS} (Transfer Characteristic)

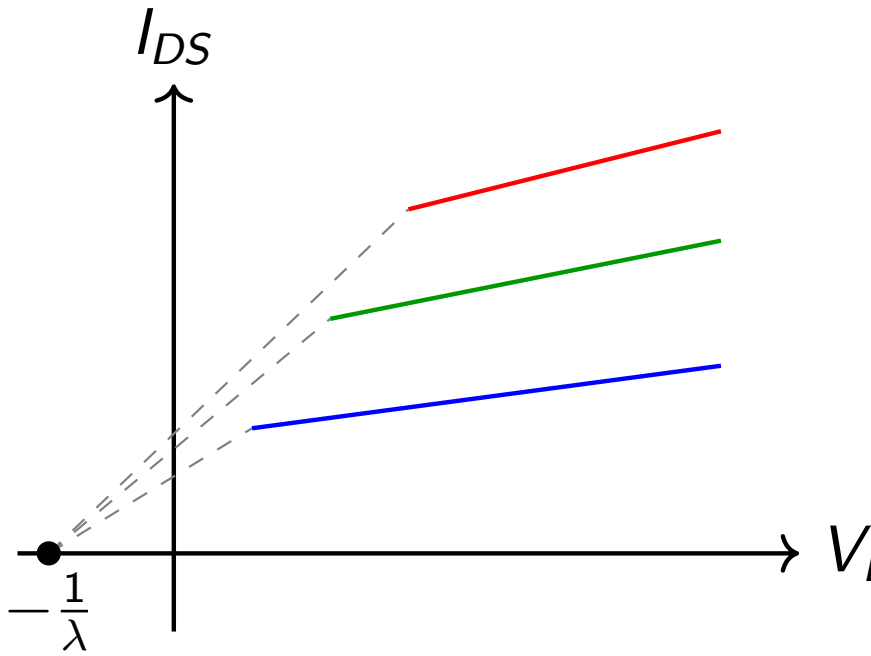


(Sec. 6.2.3) Channel-Length Modulation

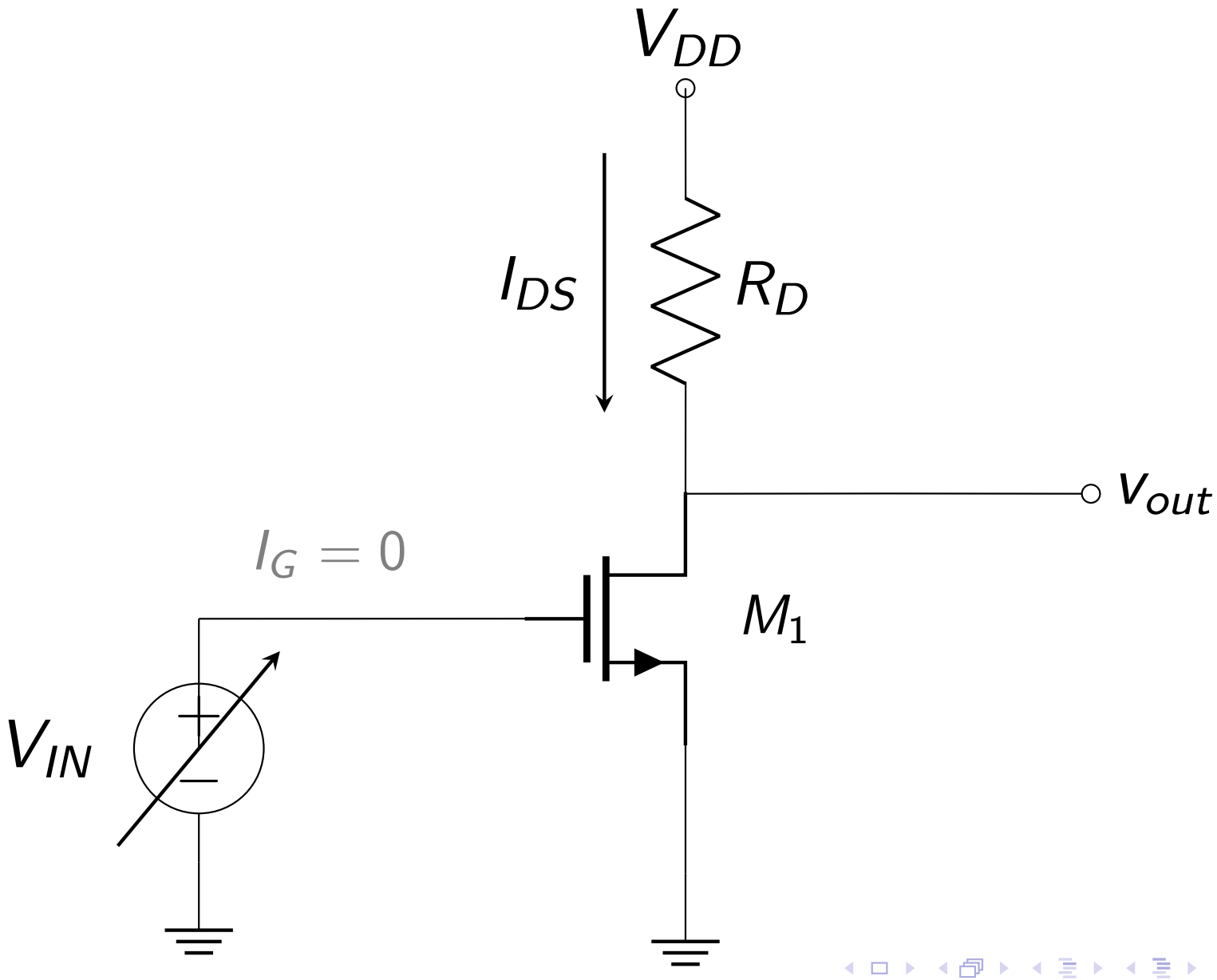
In saturation, I_{DS} is **not perfectly constant** — it increases slightly with V_{DS}:

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$$

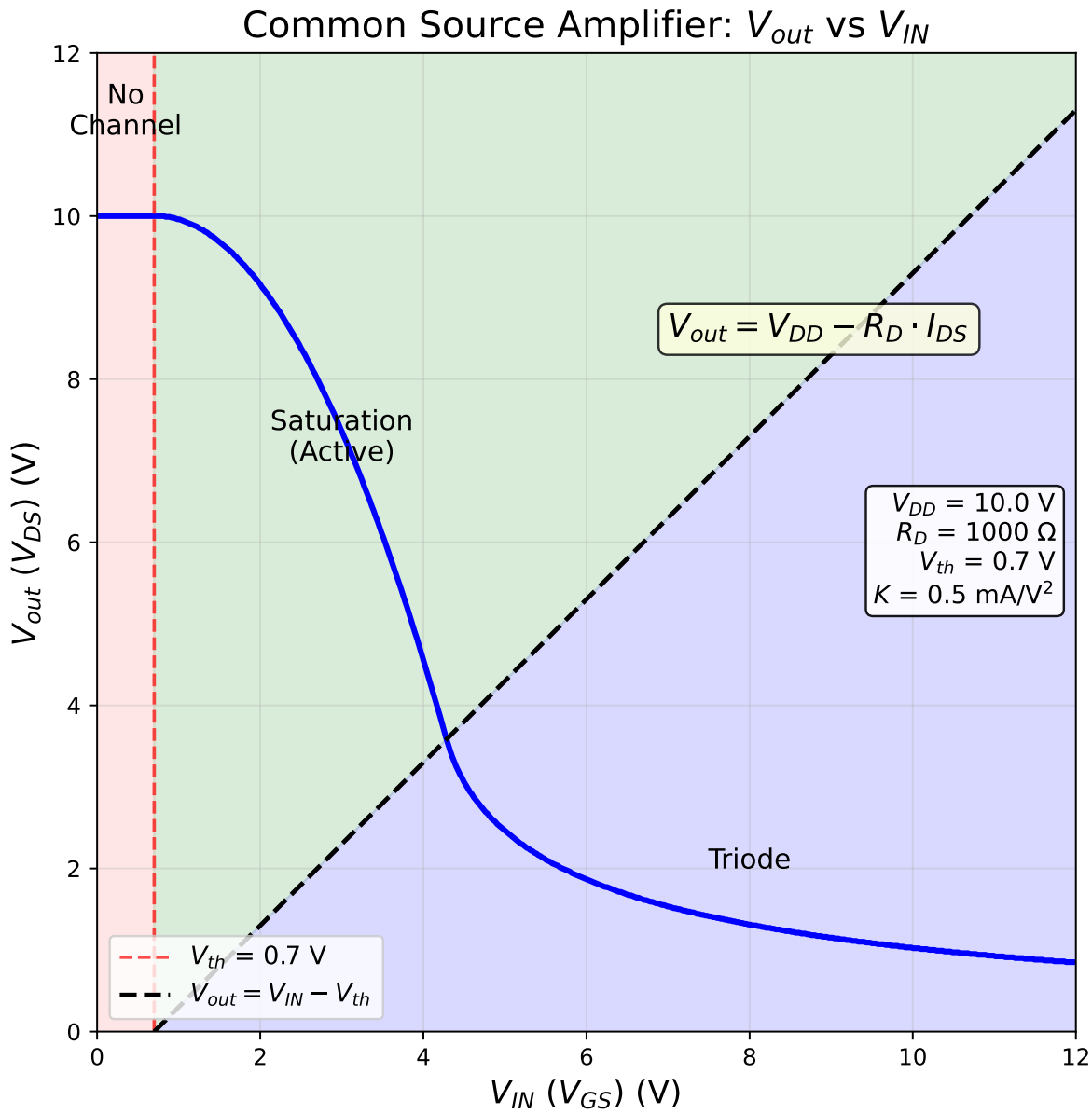
- λ = channel length modulation parameter (1/V)
- Similar to Early effect in BJT
- Causes finite output resistance: $r_o = \frac{1}{\lambda I_{DS}}$
- Extrapolating I_{DS} curves backward, they meet at $V_{DS} = -1/\lambda$



(Sec. 7.2) Common Source Amplifier



(Sec. 7.2) Common Source Amplifier – V_{out} vs V_{IN}



(Sec. 7.2) Deriving V_{out} vs V_{IN} by Hand

- **Starting point** (KVL through R_D, always true):

$$\boxed{V_{out} = V_{DD} - R_D \cdot I_{DS}}$$

- Since the source is grounded: $V_{GS} = V_{IN}$ and $V_{DS} = V_{out}$
- Choose I_{DS} based on the operating region:

Region	**Condition**	$\boldsymbol{I_{DS}}$
No Channel	$V_{IN} < V_{TN}$	$I_{DS} = 0$
Saturation	$V_{out} \geq V_{IN} - V_{TN}$	$I_{DS} = \frac{K_n}{2}(V_{IN} - V_{TN})^2$
Triode	$V_{out} < V_{IN} - V_{TN}$	$I_{DS} = \frac{K_n}{2}\left[2(V_{IN} - V_{TN})V_{out} - V_{out}^2\right]$

$\Longrightarrow$ **Pick the correct I_{DS}, then substitute into the load line.**

(Sec. 7.2) Substituting into the Load Line

Plug each I_{DS} into $V_{out} = V_{DD} - R_D \cdot I_{DS}$:

- **No Channel** ($V_{IN} < V_{TN}$):

$$V_{out} = V_{DD} \quad \text{(flat plateau)}$$

- **Saturation** ($V_{out} \geq V_{IN} - V_{TN}$):

$$V_{out} = V_{DD} - R_D \cdot \frac{K_n}{2}(V_{IN} - V_{TN})^2$$

  Explicit function of V_{IN} (downward parabola)

- **Triode** ($V_{out} < V_{IN} - V_{TN}$):

$$V_{out} = V_{DD} - R_D\, K_n \left[2(V_{IN} - V_{TN})V_{out} - V_{out}^2\right]$$

  Implicit in V_{out} — must solve numerically or iterate

Key takeaway: Saturation gives a clean, explicit $V_{out}(V_{IN})$ — this is why saturation is the useful amplification region.

(Sec. 6.2.2) Cutoff Region

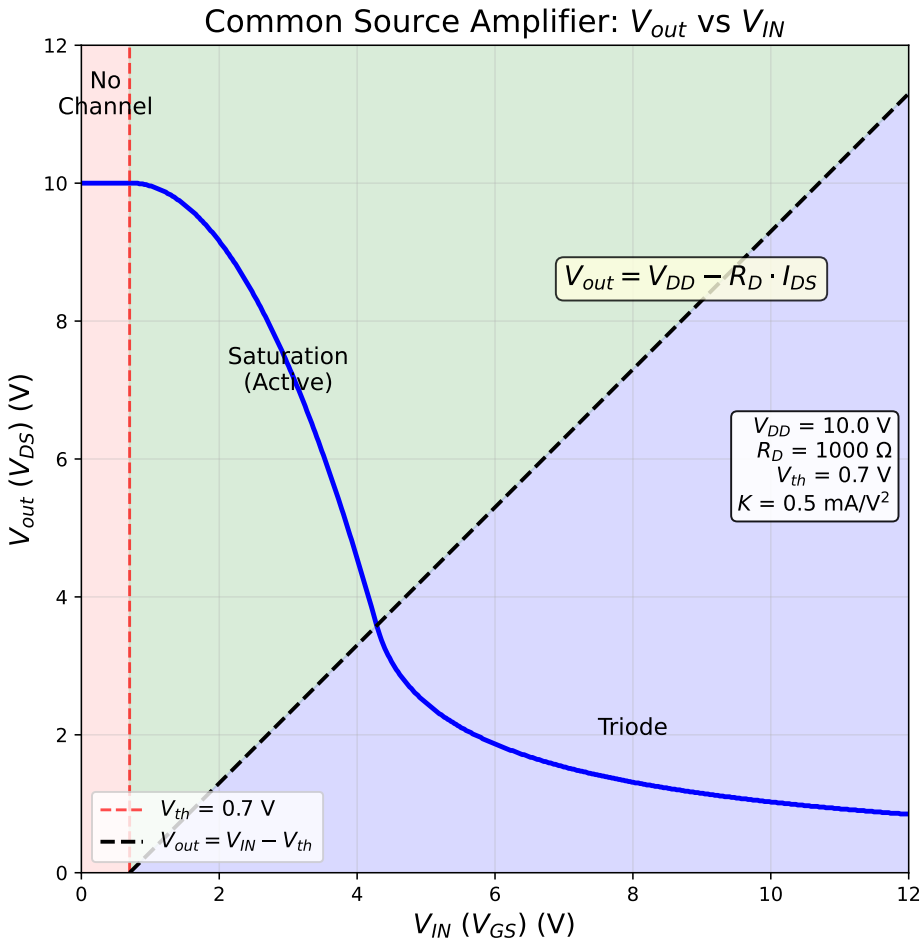
- $V_{IN} < V_{TN}$ (e.g., < 0.7 V)
- No channel forms
- $I_{DS} = 0$

Substitute into the load line:

$$V_{out} = V_{DD} - R_D \cdot I_{DS}$$
$$= V_{DD} - R_D \cdot 0$$

$$\boxed{V_{out} = V_{DD}}$$

Output stays flat at V_{DD}
(transistor is OFF)



(Sec. 6.2.2) Saturation Region (Active)

- $V_{IN} > V_{TN}$ and $V_{out} \geq V_{IN} - V_{TN}$
- Channel is pinched off at drain
- $I_{DS} = \frac{K_n}{2}(V_{IN} - V_{TN})^2$

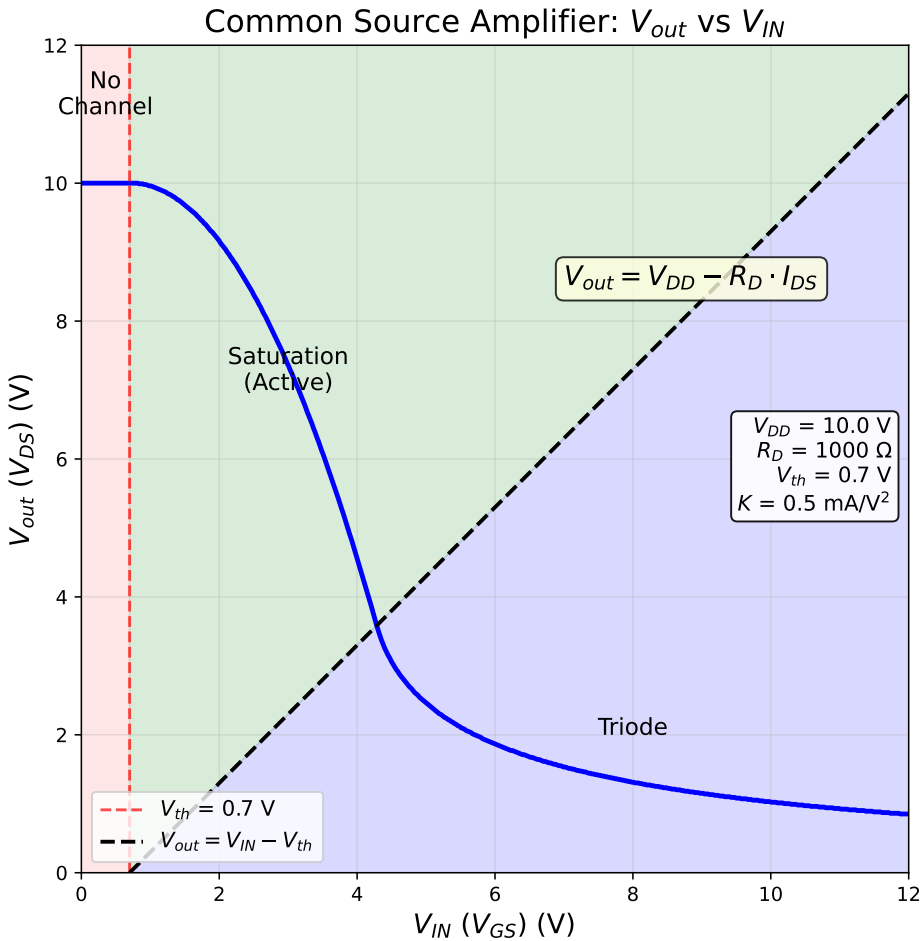
Substitute into the load line:

$$V_{out} = V_{DD} - R_D \cdot I_{DS}$$

$$= V_{DD} - R_D \cdot \frac{K_n}{2}(V_{IN} - V_{TN})^2$$

$$\boxed{V_{out} = V_{DD} - R_D \cdot \frac{K_n}{2}(V_{IN} - V_{TN})^2}$$

Explicit function of V_{IN} (downward parabola)



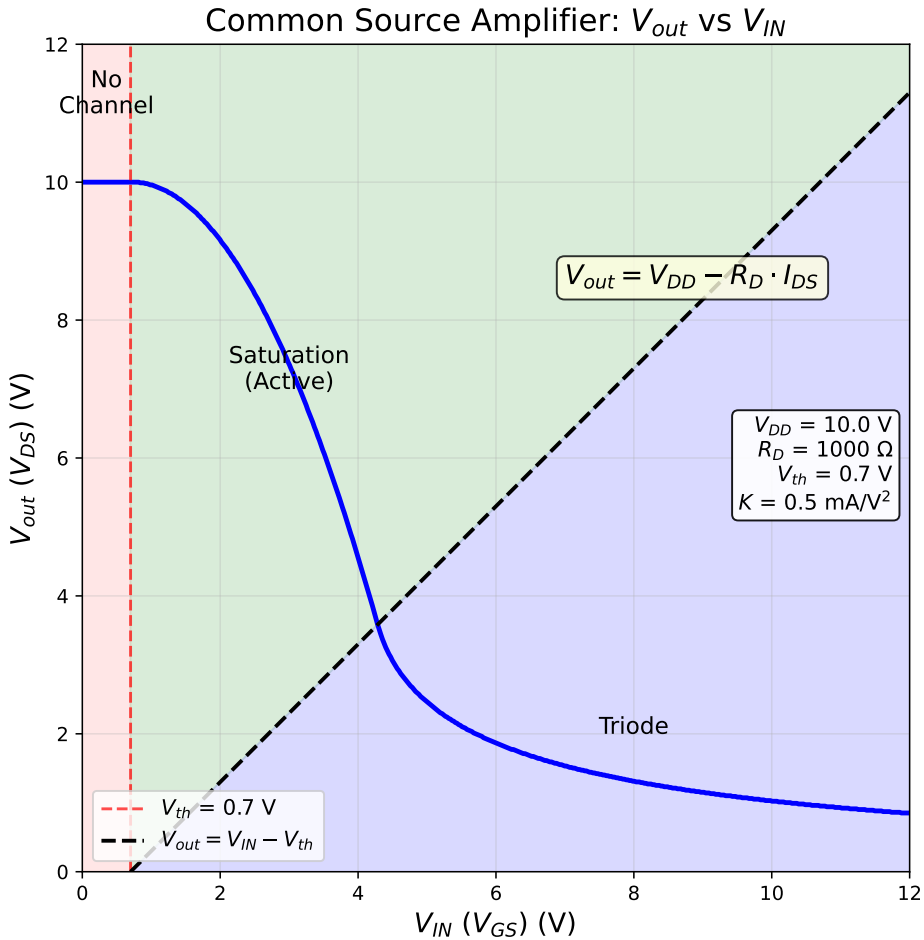
(Sec. 6.2.2) Triode (Linear) Region

- $V_{IN} > V_{TN}$ and $V_{out} < V_{IN} - V_{TN}$
- $I_{DS} = \frac{K_n}{2}\left[2(V_{IN} - V_{TN})V_{out} - V_{out}^2\right]$

Substitute into the load line:

$$V_{out} = V_{DD} - R_D \cdot I_{DS}$$

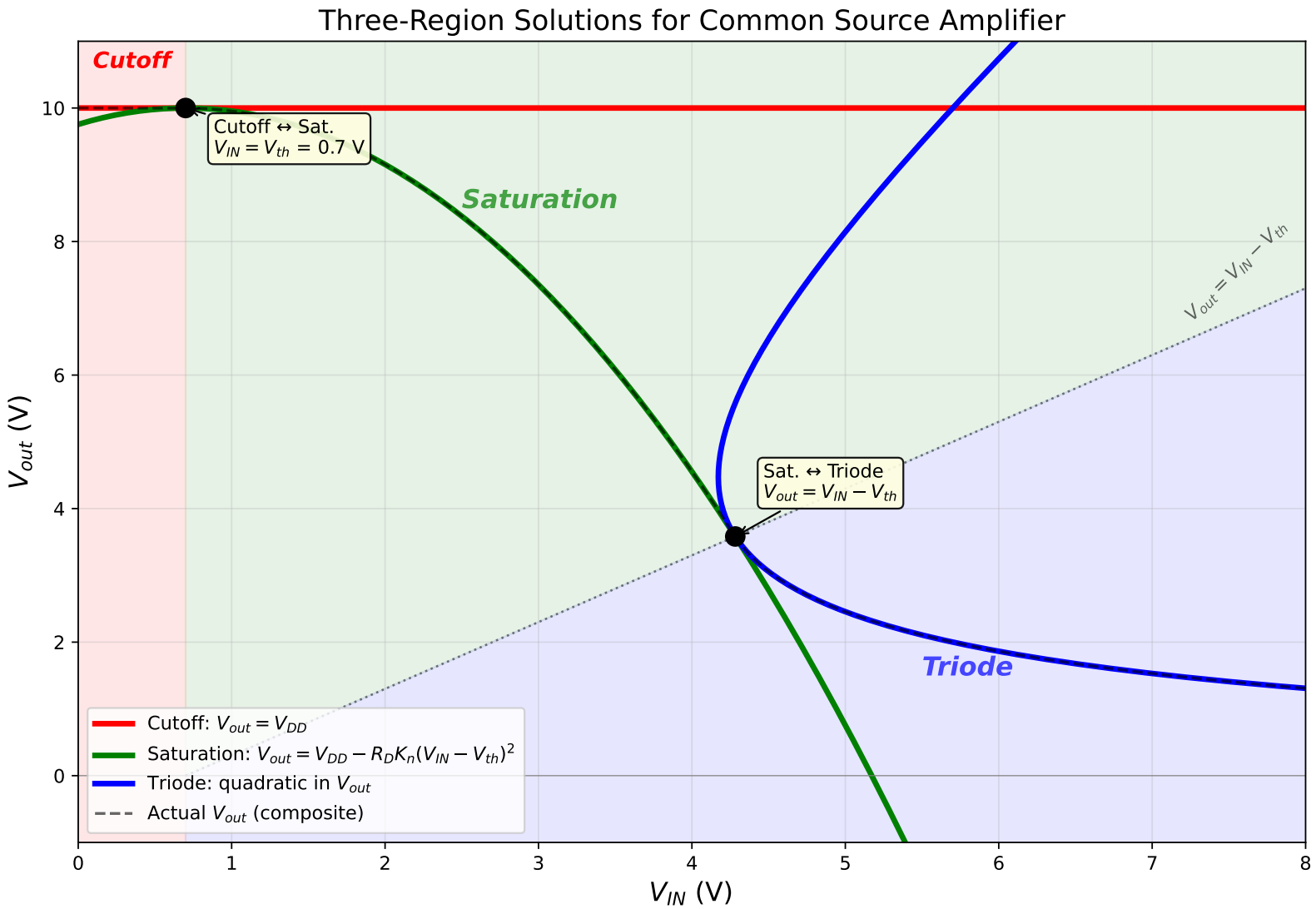
$$= V_{DD} - R_D \cdot \frac{K_n}{2}\left[2(V_{IN} - V_{TN})V_{out} - V_{out}^2\right]$$



Quadratic in V_{out} (solvable)

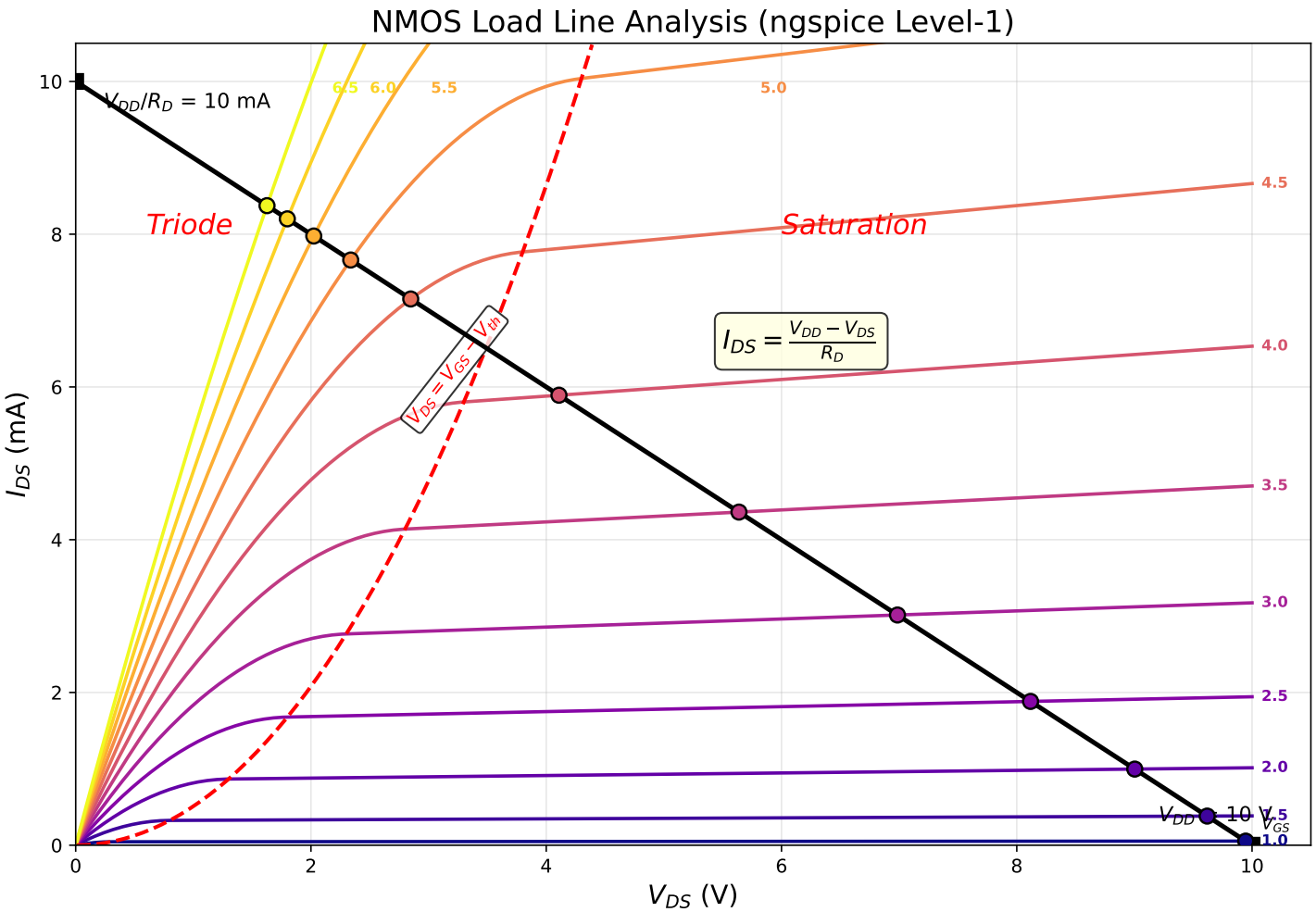
Output levels off (MOSFET is fully ON)

Three-Region Solutions for V_{out} vs V_{IN}

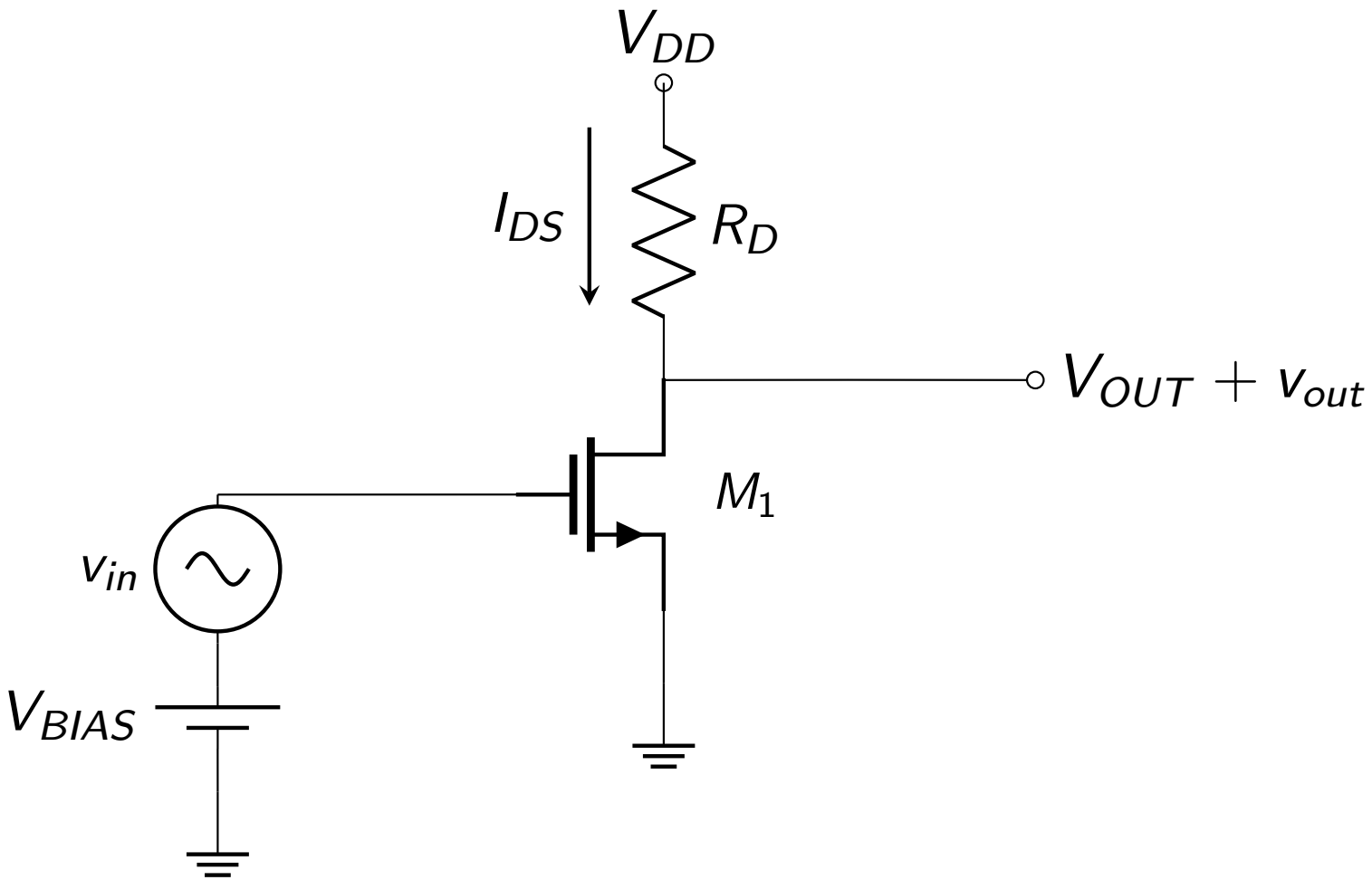


(Sec. 7.2) Load Line Analysis

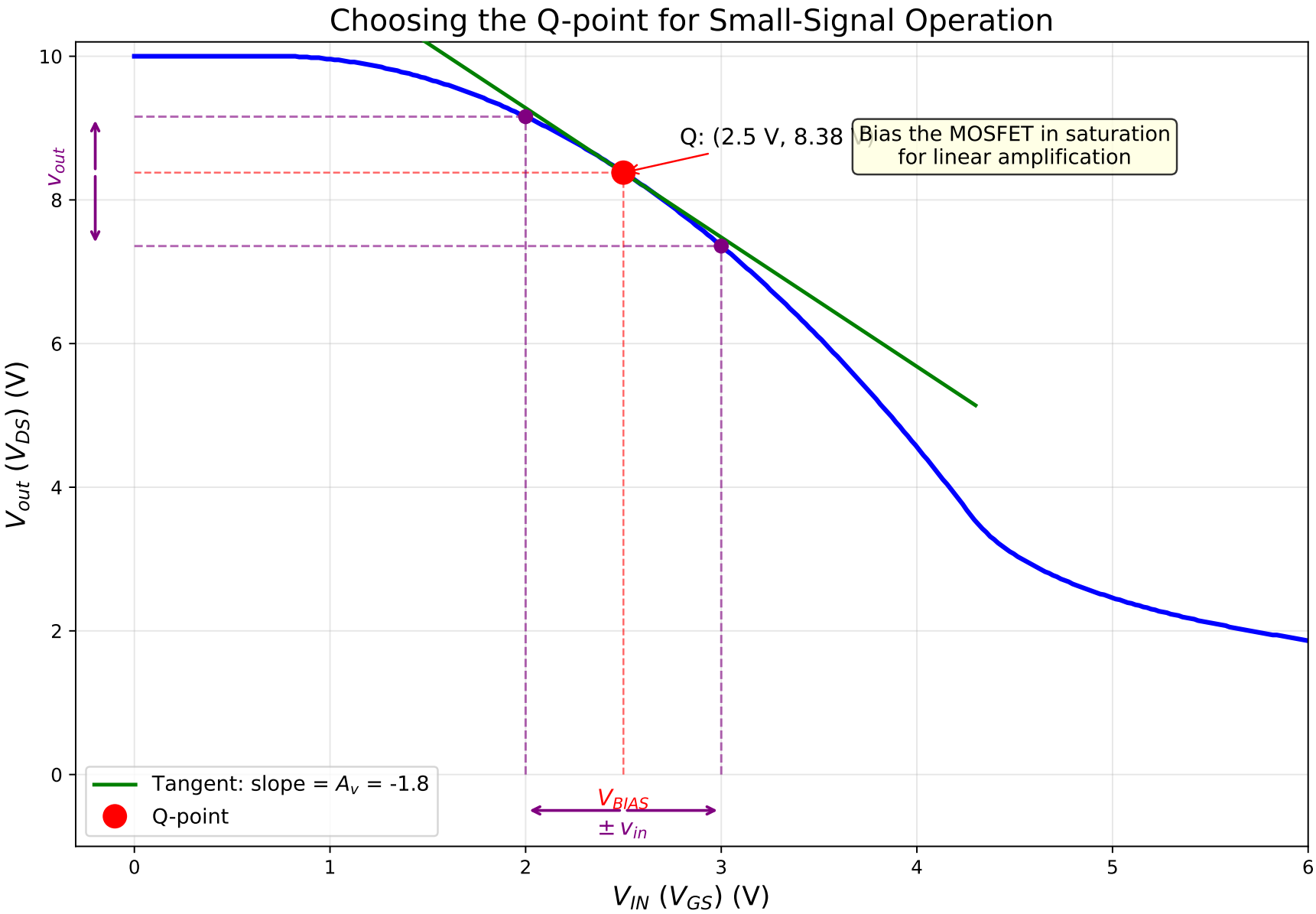
Load line equation: $$I_{DS} = \frac{V_{DD} - V_{DS}}{R_D}$$



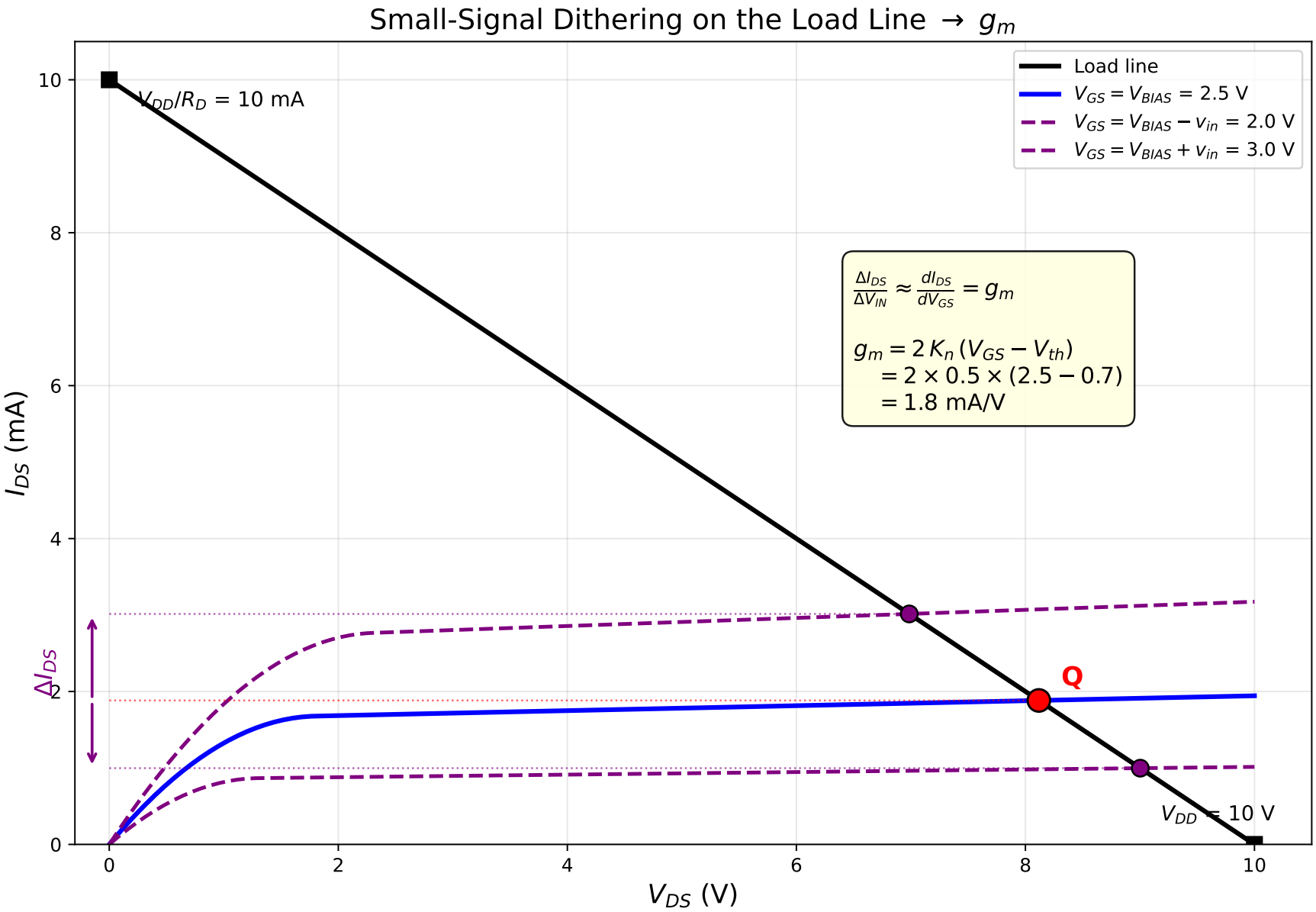
(Sec. 7.2) Common Source Amplifier with Small-Signal Input



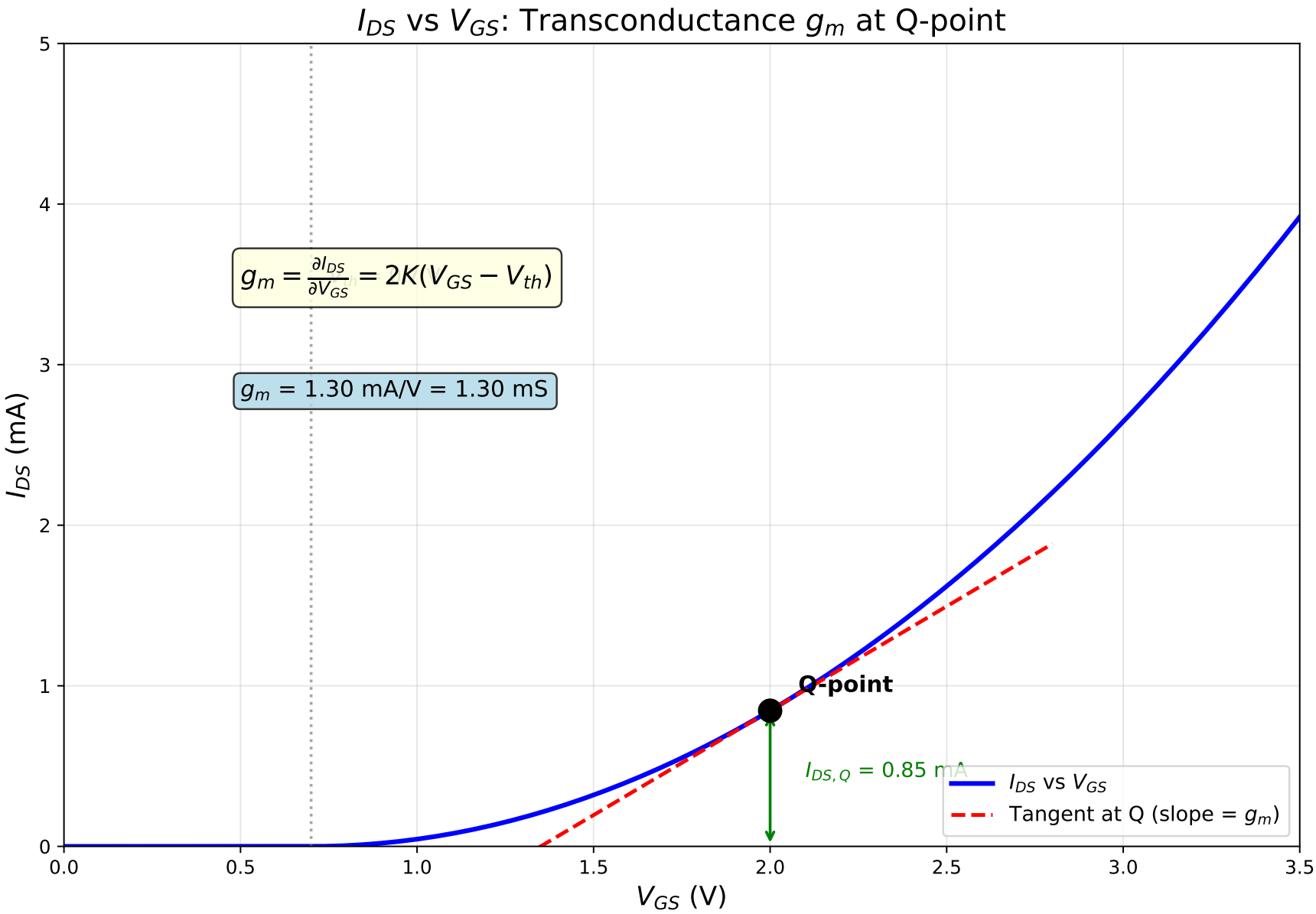
(Sec. 7.2) V_{out} vs V_{IN} — Choosing a Q-point



(Sec. 7.2.3) From Load Line Dithering to g_m



(Sec. 7.2.3) I_{DS} vs V_{GS}: Linearization at the Q-point



(Sec. 7.2.3) Transconductance g_m for MOSFET

Linearizing I_{DS} vs V_{GS} at the Q-point:

$$\boxed{g_m = \left.\frac{\partial I_{DS}}{\partial V_{GS}}\right|_Q = K_n(V_{GS} - V_{TN}) = \frac{2I_{DS}}{V_{GS} - V_{TN}}}$$

g_m tells us: *how much does I_{DS} change for a small change in V_{GS}?*

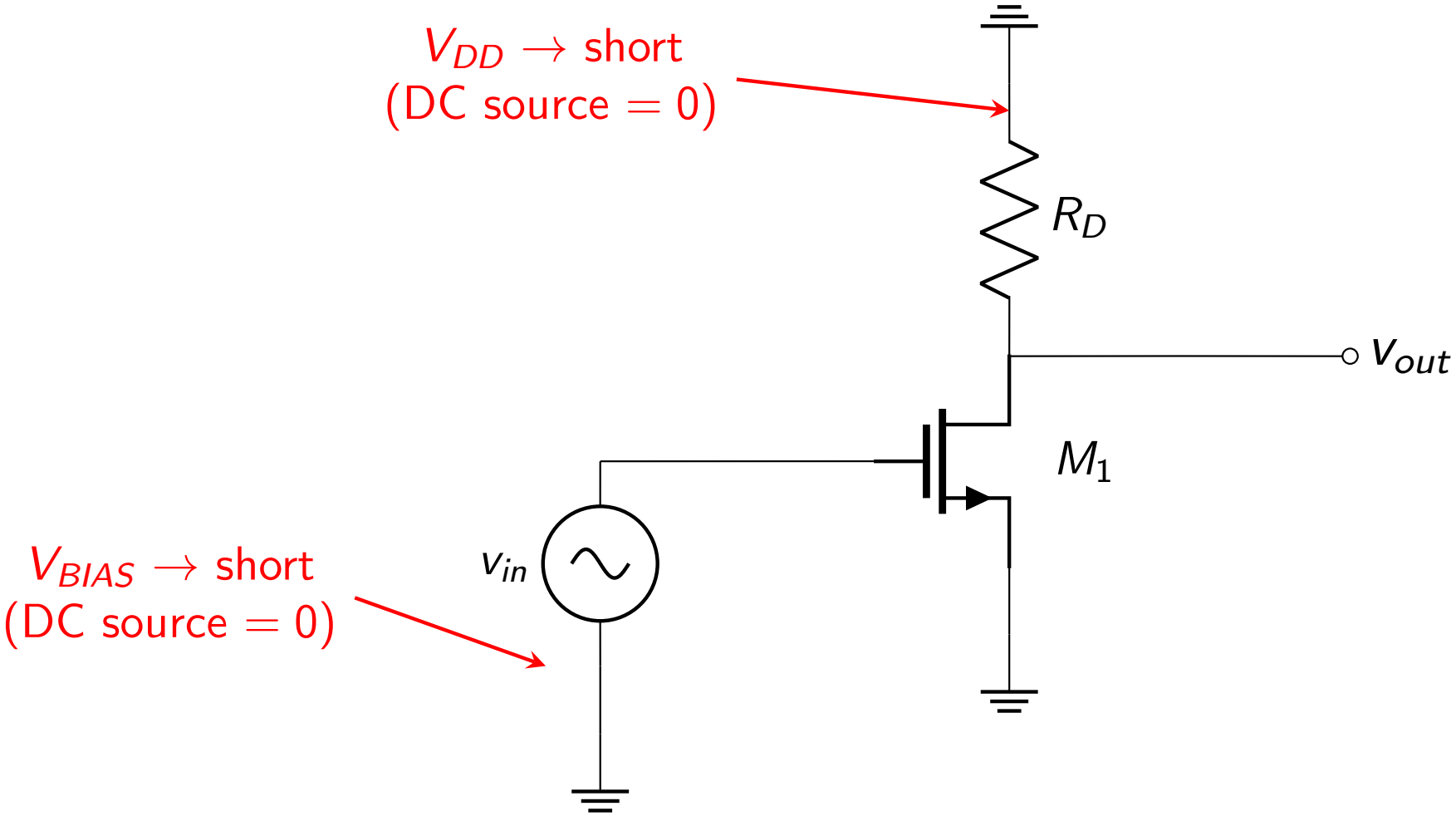
Comparison with BJT:

	BJT	**MOSFET**
g_m formula	$\frac{I_C}{V_T}$	$\frac{2I_{DS}}{V_{GS} - V_{TN}}$
Typical g_m	Higher (exponential)	Lower (square-law)

For the same current, BJT typically has higher g_m because $V_T \approx 26$ mV $\ll V_{GS} - V_{TN}$.

(Sec. 7.2.4) Shorting DC Sources for Small-Signal Analysis

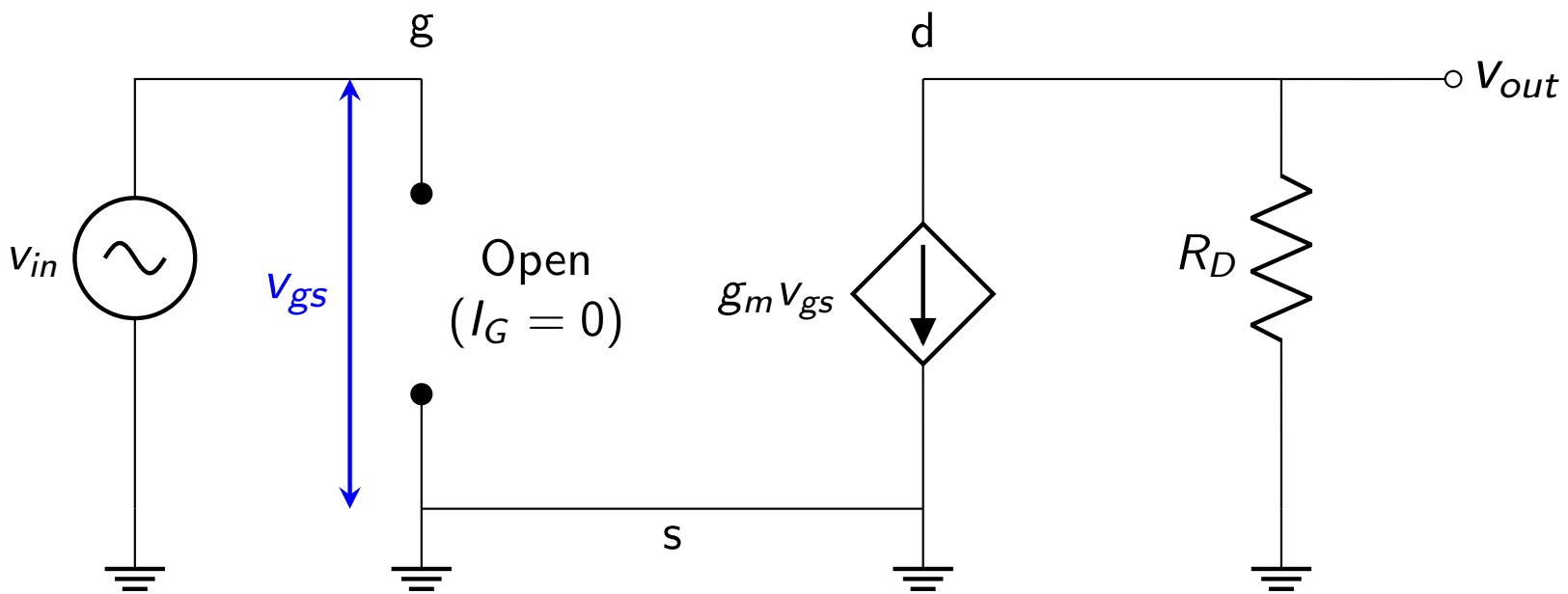
To analyze small-signal behavior, replace all DC voltage sources with short circuits:



Next step: replace the MOSFET with its small-signal model

(Sec. 7.2.4) Small-Signal Equivalent Circuit

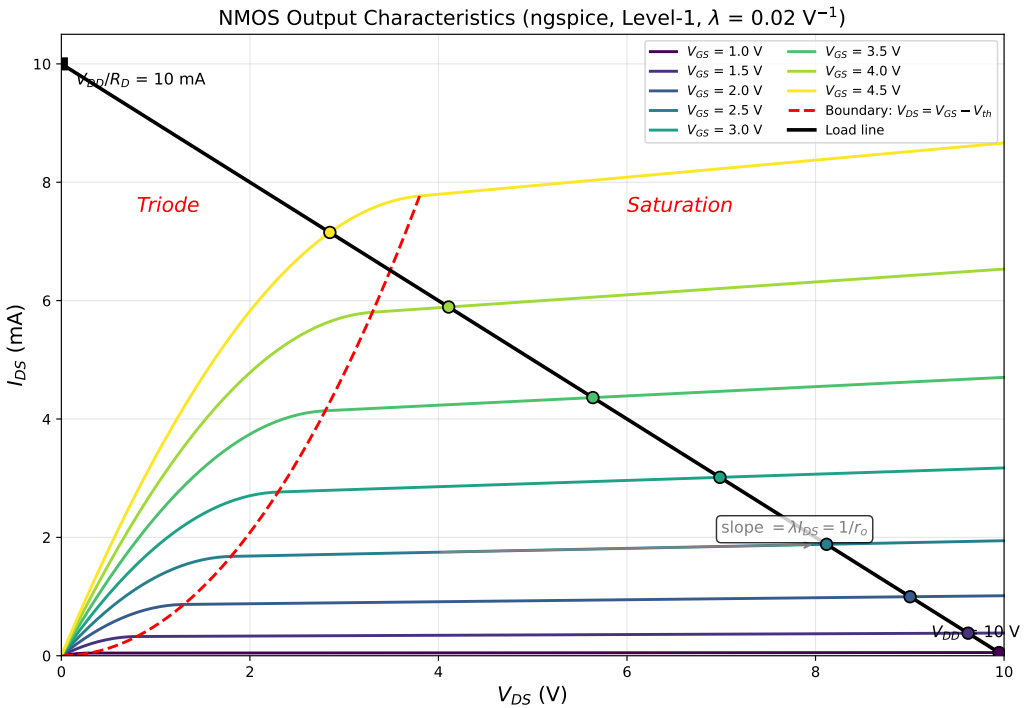
Short all DC sources ($V_{BIAS} \rightarrow$ short, $V_{DD} \rightarrow$ short) and replace MOSFET with small-signal model:



$$v_{gs} = v_{in} \qquad v_{out} = -g_m\, v_{gs} \cdot R_D$$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = -g_m R_D}$$

(Sec. 7.2.4) Channel-Length Modulation ⇒ Output Resistance r_o



With CLM, I_{DS} depends on V_{DS}:

$$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$$

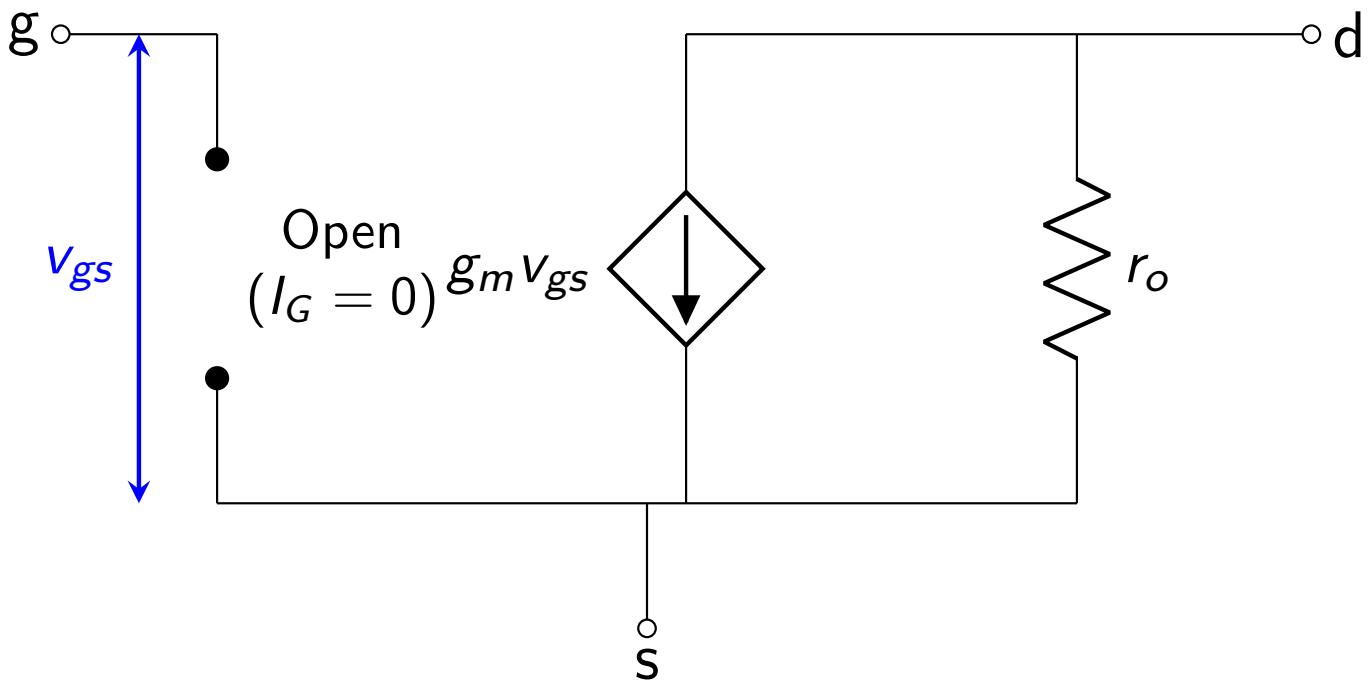
The saturation curves have a slight **upward slope**:

$$\left.\frac{\partial I_{DS}}{\partial V_{DS}}\right|_Q = \lambda I_{DS} \Rightarrow \boxed{r_o = \frac{1}{\lambda I_{DS}}}$$

Red dashed line: triode–saturation boundary ($V_{DS} = V_{GS} - V_{TN}$)

r_o acts as a **resistor** in parallel with the current source $g_m v_{gs}$.

(Sec. 7.2.4) Complete Small-Signal Model of the NMOS



$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TN}} = K_n(V_{GS} - V_{TN})$$

$$r_o = \frac{1}{\lambda I_{DS}}$$

$$A_v = \frac{v_{out}}{v_{in}} = -g_m\,(R_D \| r_o) = -g_m \frac{R_D\, r_o}{R_D + r_o}$$

(Sec. 7.2) BJT vs MOSFET: Small-Signal Comparison

Parameter	**BJT (npn)**	**NMOS MOSFET**
Input impedance	$r_\pi = \beta/g_m$ (finite)	∞ (gate is insulated)
Transconductance	$g_m = I_C/V_T$	$g_m = 2I_{DS}/(V_{GS} - V_{TN})$
Output resistance	$r_o = V_A/I_C$	$r_o = 1/(\lambda I_{DS})$
Voltage gain	$A_v = -g_m(R_C \| r_o)$	$A_v = -g_m(R_D \| r_o)$
Input current	$i_b = v_{be}/r_\pi$	$i_{gs} = 0$

Typical Values of $\mu_n C_{ox}$ for Lecture Examples

The **process transconductance parameter** $k'_n \equiv \mu_n C_{ox}$ is set by fabrication (μ_n and t_{ox}), not by the designer.

Defensible classroom ranges for NMOS:

Regime	$\mu_n C_{ox}$
Conservative (older, thick-oxide process)	$\approx 70\,\mu\text{A/V}^2$
Typical textbook (0.25–0.18 μm)	100–200 $\mu\text{A/V}^2$
Aggressive but still reasonable for hand analysis	250–300 $\mu\text{A/V}^2$

For EE461 we use $\mu_n C_{ox} = 200\,\mu\text{A/V}^2$ — a clean mid-range value for hand analysis.

Supporting data: a measured 1.6 μm CMOS process (gate oxide 306 Å) reports $K' = \mu_n C_{ox}/2 = 35.7\,\mu\text{A/V}^2$, i.e. $\mu_n C_{ox} \approx 71\,\mu\text{A/V}^2$. Modern short-channel processes push $\mu_n C_{ox}$ well above 200 $\mu\text{A/V}^2$.

Notation Warning: k_n' vs K'

Different textbooks split the factor of $\frac{1}{2}$ differently — check which convention your source uses before quoting a number:

Convention A (Razavi, Sedra/Smith):

$$I_D = \tfrac{1}{2} \underbrace{\mu_n C_{ox}}_{k_n'} \tfrac{W}{L} V_{OV}^2$$

Report $k_n' = \mu_n C_{ox}$ directly (∼100–300 μA/V^2).

Convention B (older books, some datasheets):

$$I_D = K' \tfrac{W}{L} V_{OV}^2, \quad K' = \tfrac{\mu_n C_{ox}}{2}$$

Reported K' is **half** of k_n' (∼50–150 μA/V^2).

Mixing the two conventions gives a **factor-of-two error** in I_D. Always check which one is in play.

EE461 equation sheet convention: $K_n = \mu_n C_{ox} \cdot (W/L)$ and $I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$ — equivalent to Convention A with W/L pulled out.

Where $\mu_n C_{ox}$ Comes From

$$k_n' = \mu_n C_{ox} = \mu_n \cdot \frac{\varepsilon_{ox}}{t_{ox}}.$$

Two process-dependent pieces:

- μ_n — electron mobility in the channel (typ. 400–600 $\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ in long-channel devices)
- $C_{ox} = \varepsilon_{ox}/t_{ox}$ — gate-oxide capacitance per unit area ($\varepsilon_{ox} = 3.9\,\varepsilon_0 \approx 3.45 \times 10^{-11}$ F/m)

Example. For $t_{ox} = 10\,\mathrm{nm}$ and $\mu_n = 500\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$:

$$C_{ox} = \frac{3.45 \times 10^{-11}}{10^{-8}} \approx 3.45\,\mathrm{mF/m}^2$$

$$\mu_n C_{ox} \approx (0.05\,\mathrm{m}^2/\mathrm{V}\cdot\mathrm{s})(3.45 \times 10^{-3}\,\mathrm{F/m}^2) \approx 170\,\mu\mathrm{A/V}^2.$$

Thinner oxide $\Rightarrow$ larger C_{ox} $\Rightarrow$ larger $\mu_n C_{ox}$. Newer processes have thinner oxides, hence higher k_n'.