EE461g: Introduction to Electronic Circuits

Lecture 15

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Outline

Measuring Input and Output Impedances (Razavi, Fig. 5.2)

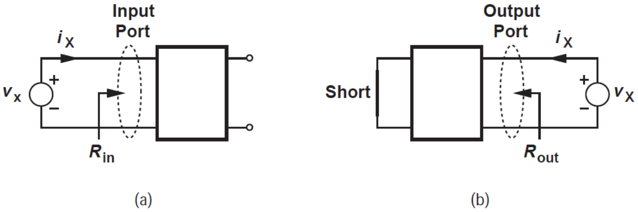
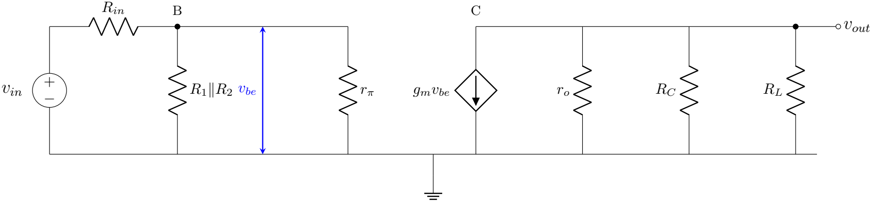


Figure 5.2 Measurement of (a) input and (b) output impedances.

Review
CE Small-Signal Model

CE Small-Signal Equivalent Circuit



$R_1 \| R_2$ at input, r_π from base to emitter, $g_m v_{be}$ controlled source, $r_o \| R_C \| R_L$ at output

Two-Port Parameters We Need

Every amplifier can be characterized as a **two-port network** with three parameters:

1. **Input impedance** Z_{in}: what the source "sees" looking into the amplifier
2. **Output impedance** Z_{out}: what the load "sees" looking back into the amplifier
3. **Open-circuit voltage gain** A_{vo}: gain with no load ($R_L = \infty$)

The overall voltage gain is:

$$\boxed{A_v = \frac{Z_{in}}{R_{in} + Z_{in}} \cdot A_{vo} \cdot \frac{R_L}{Z_{out} + R_L}}$$

Today: a systematic method to **derive** Z_{in} and Z_{out} for any circuit

The Test Source Method

What is Input Impedance?

Z_{in} is the resistance the source "sees" looking into the amplifier input port.

Procedure to find Z_{in}:

1. **Remove** the signal source (v_{in} and R_{in})
2. **Attach** an imaginary ideal voltage source v_x at the input port
3. **Find** the current i_x drawn from v_x
4. **Compute:**

$$\boxed{Z_{in} = \frac{v_x}{i_x}}$$

Think of it as: "If I push voltage v_x into this port, how much current i_x flows?"
More current $\Rightarrow$ lower impedance.

What is Output Impedance?

Z_{out} is the resistance the load "sees" looking back into the amplifier output port.

Procedure to find Z_{out}:

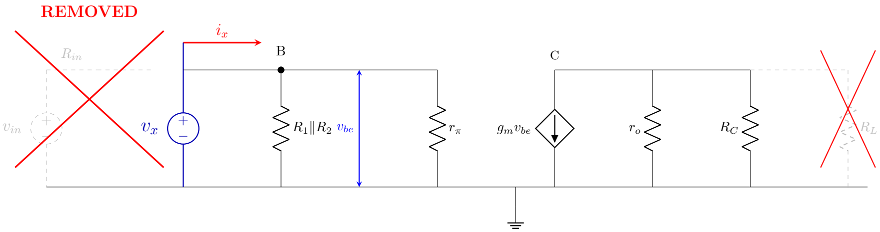
1. **Remove** the source: set $v_{in} = 0$ and remove R_{in} (short the input port)
2. **Remove** the load R_L
3. **Attach** an ideal voltage source v_x at the output port
4. **Find** the current i_x flowing into v_x
5. **Compute:**

$$\boxed{Z_{out} = \frac{v_x}{i_x}}$$

Removing the source (shorting the input port) often turns off the controlled source,
which greatly simplifies the circuit.

Finding Z_{in} for the CE Amplifier

CE Z_{in} Test Circuit

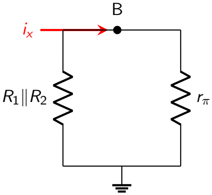


v_x replaces the signal source at the base node. i_x flows into the circuit.
R_L is removed (we are characterizing the amplifier itself).

KCL at the base node: i_x splits into two parallel paths to ground:

1. Current through $R_1 \| R_2$:
$$i_1 = \frac{v_x}{R_1 \| R_2}$$

2. Current through r_π: $\quad i_2 = \frac{v_x}{r_\pi}$



By KCL:

$$i_x = i_1 + i_2 = \frac{v_x}{R_1 \| R_2} + \frac{v_x}{r_\pi}$$

Therefore:

$$\boxed{Z_{in} = \frac{v_x}{i_x} = R_1 \| R_2 \| r_\pi}$$

Razavi, Eq. 5.224, p. 219

CE Z_{in} — Numerical Example

Using values from Lecture 12: $R_1 = 200\ \text{k}\Omega$, $R_2 = 100\ \text{k}\Omega$, $\beta = 100$, $I_C = 3.96$ mA

Step 1: Find r_π:

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{100 \times 0.026}{0.00396} = 657\ \Omega$$

Step 2: Compute Z_{in}:

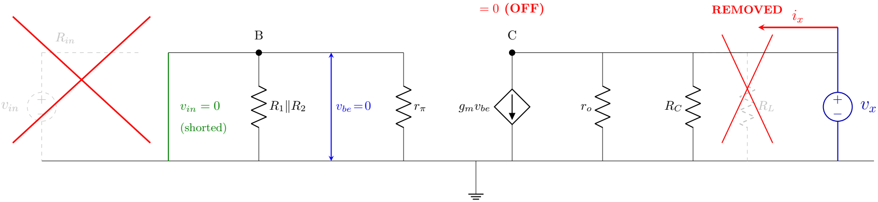
$$R_1 \| R_2 = \frac{200\text{k} \times 100\text{k}}{200\text{k} + 100\text{k}} = 66.7\ \text{k}\Omega$$

$$Z_{in} = R_1 \| R_2 \| r_\pi = 66.7\ \text{k}\Omega \| 657\ \Omega \approx \boxed{650\ \Omega}$$

$r_\pi = 657\ \Omega$ dominates because it is **much smaller** than $R_1 \| R_2 = 66.7\ \text{k}\Omega$.
The small r_π "pulls down" the input impedance of the CE amplifier.

Finding Z_{out} for the CE Amplifier

CE Z_{out} Test Circuit

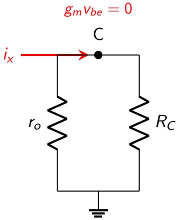


- Source removed ($v_{in} = 0$, R_{in} removed) $\Rightarrow$ base shorted to ground
- Since both ends of r_π are at ground: $v_{be} = 0$
- Therefore $g_m v_{be} = g_m \cdot 0 = 0$ — **controlled source turns off!**
- v_x is applied at the collector; R_L is removed

CE Z_{out} — Step-by-Step

With the controlled source off, i_x splits into two parallel paths:

1. Current through r_o: $\quad i_1 = \dfrac{v_x}{r_o}$
2. Current through R_C: $\quad i_2 = \dfrac{v_x}{R_C}$



By KCL:

$$i_x = \frac{v_x}{r_o} + \frac{v_x}{R_C}$$

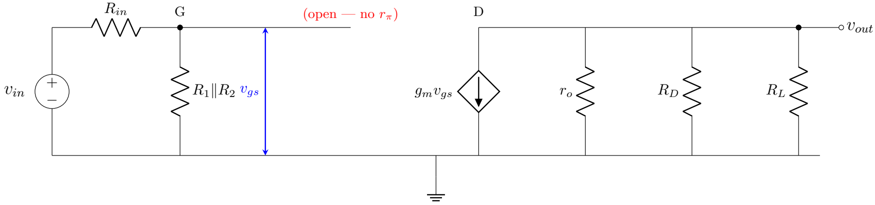
$$\boxed{Z_{out} = \frac{v_x}{i_x} = r_o \| R_C}$$

With no Early effect ($V_A = \infty \Rightarrow r_o = \infty$): $Z_{out} = \infty \| R_C = R_C = \boxed{1\text{ k}\Omega}$

Razavi, Eq. 5.218, p. 217

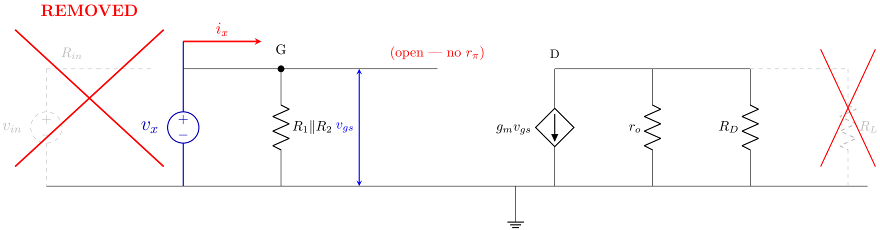
Finding Z_{in} for the CS Amplifier

CS Small-Signal Equivalent Circuit (Review)



Key difference from CE: No r_π between gate and source. The MOSFET gate draws **zero current** — infinite input impedance at the gate itself.

CS Z_{in} Test Circuit



- v_x replaces the signal source at the gate node
- The gate is an **open circuit** — no current flows into the MOSFET gate
- i_x can **only** flow through $R_1 \| R_2$ to ground

$$\boxed{Z_{in} = \frac{v_x}{i_x} = R_1 \| R_2}$$

Razavi, Eq. 7.79, p. 318

CS Z_{in} — Numerical Example

Using values from Lecture 12:

$$R_1 = 400\ \text{k}\Omega, \qquad R_2 = 100\ \text{k}\Omega$$

$$Z_{in} = R_1 \| R_2 = \frac{400\text{k} \times 100\text{k}}{400\text{k} + 100\text{k}} = \boxed{80\ \text{k}\Omega}$$

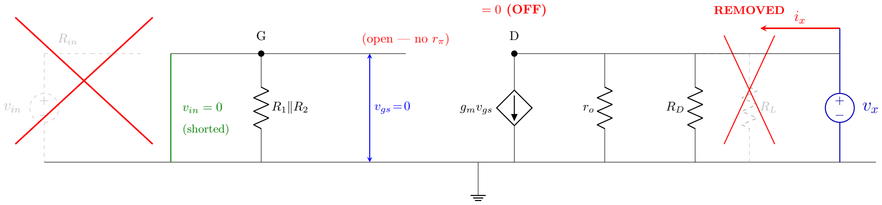
Compare to CE:

	CE (BJT)	CS (MOSFET)
Z_{in}	650 Ω	80 kΩ

The CS amplifier has $\sim 123\times$ higher input impedance!
This is because the MOSFET gate draws no current
(no r_π to "pull down" Z_{in}).

Finding Z_{out} for the CS Amplifier

CS Z_{out} Test Circuit



- Source removed ($v_{in} = 0$, R_{in} removed) $\Rightarrow$ gate shorted to ground $\Rightarrow$ $v_{gs} = 0 \Rightarrow$ controlled source off
- Remove R_L, attach v_x at drain
- Same logic as CE: i_x splits into r_o and R_D in parallel

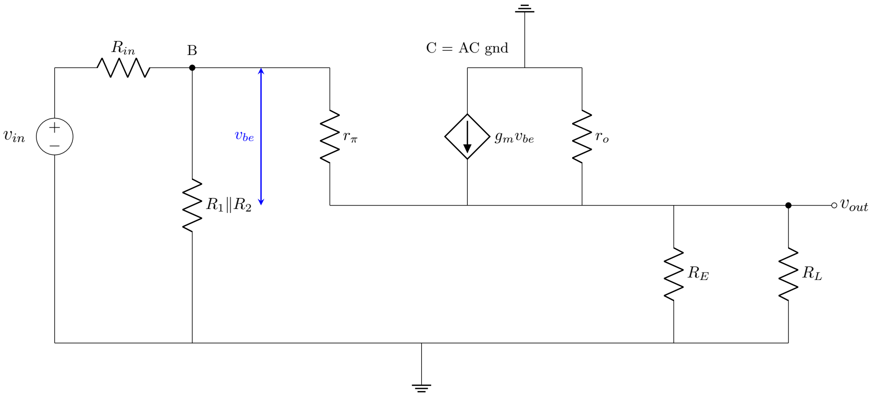
$$\boxed{Z_{out} = \frac{v_x}{i_x} = r_o \| R_D}$$

With $\lambda = 0$ ($r_o = \infty$): $\quad Z_{out} = R_D = \boxed{1\ \text{k}\Omega}$

Razavi, §7.2–7.3; analogous to CE (Eq. 5.218)

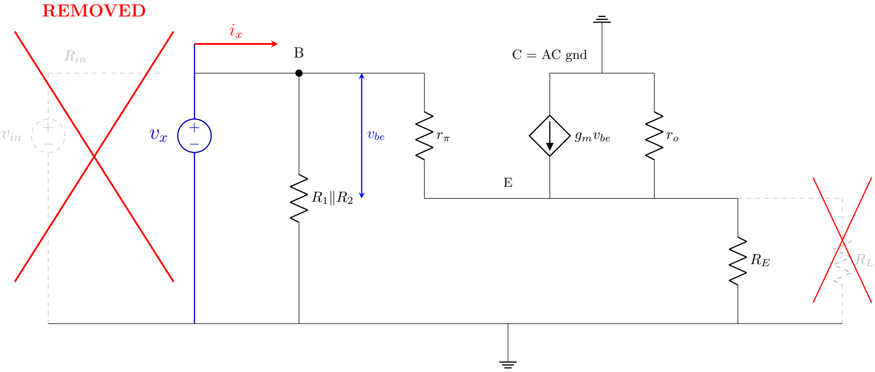
Finding Z_{in} for the CC Amplifier

CC Small-Signal Equivalent Circuit (Review)



CC (Emitter Follower): Collector at AC ground (V_{CC}).
Output taken from the **emitter** through R_E.

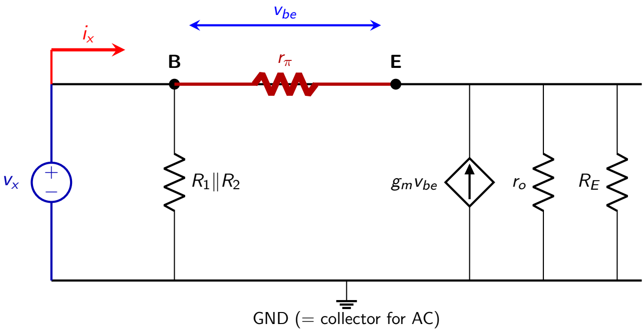
CC Z_{in} Test Circuit



- v_x replaces the signal source at the base node
- i_x flows through $R_1 \| R_2$ **and** into the base (r_π)
- Unlike CE: the emitter is **not** grounded — R_E creates feedback!

CC Z_{in} — Simplified Circuit (Collector = AC Ground)

Since the collector is at AC ground, it merges with ground. Redraw:



Now it's clear: i_x splits into $R_1 \| R_2$ (down) and r_π (across to E). Current through r_π must also exit through R_E — this is why R_E "reflects" back!

CC Z_{in} — Impedance Reflection

Key insight: Current through r_π also flows through R_E at the emitter.

KCL at the emitter node (with $r_o = \infty$):

$$i_B + g_m v_{be} = \frac{v_E}{R_E}$$

Since $i_B = v_{be}/r_\pi$ and $g_m = \beta/r_\pi$:

$$\frac{v_{be}}{r_\pi}(\beta + 1) = \frac{v_E}{R_E}$$

With $v_{be} = v_x - v_E$, solving gives:

$$i_B = \frac{v_x}{r_\pi + (\beta + 1)R_E}$$

The base "sees" R_E multiplied by $(\beta + 1)$!

This is called **impedance reflection.**

$$\boxed{Z_{in} = R_1 \| R_2 \| \left[r_\pi + (\beta + 1)R_E\right]}$$

Razavi, Eq. 5.326, p. 242

CC Z_{in} — Numerical Example

Using values from Lecture 13: $R_1 = 200\ \text{k}\Omega$, $R_2 = 100\ \text{k}\Omega$, $R_E = 1\ \text{k}\Omega$, $\beta = 100$, $I_C = 1.59$ mA

Step 1: $r_\pi = \beta V_T / I_C = 100 \times 0.026/0.00159 = 1636\ \Omega$

Step 2: Impedance looking into the base:

$$Z_{in,base} = r_\pi + (\beta + 1)R_E = 1636 + 101 \times 1000 = 102{,}636\ \Omega \approx 102.6\ \text{k}\Omega$$

Step 3: $R_1 \| R_2 = 66.7\ \text{k}\Omega$

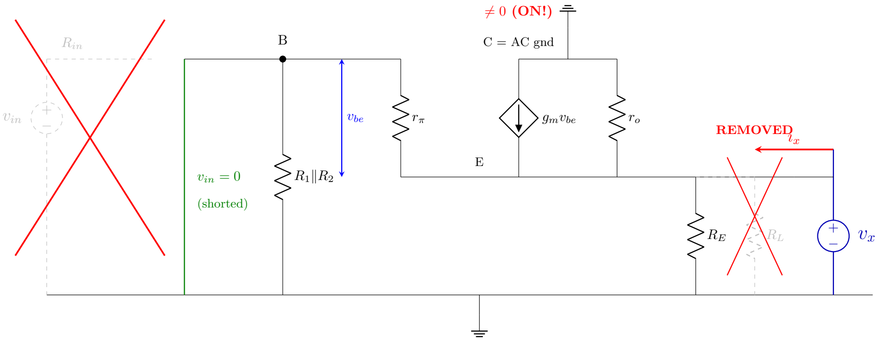
$$Z_{in} = 66.7\ \text{k}\Omega \| 102.6\ \text{k}\Omega = \boxed{40.4\ \text{k}\Omega}$$

Compare: CE has $Z_{in} = 650\ \Omega$. CC has $Z_{in} = 40.4\ \text{k}\Omega$ — **62**$\times$ **higher!**

The $(\beta + 1)R_E$ term makes the base impedance very large.

Finding Z_{out} for the CC Amplifier

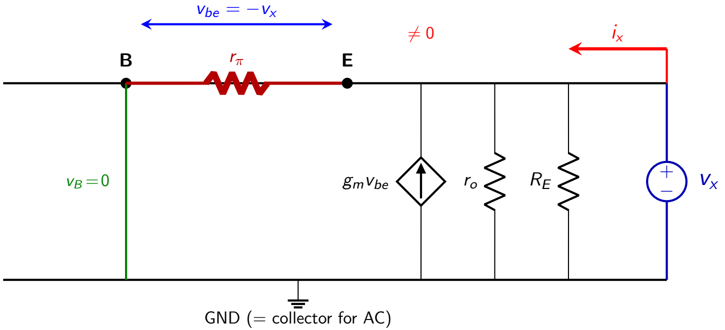
CC Z_{out} Test Circuit



- Source removed (v_{in} and R_{in}) — input port shorted $\Rightarrow$ base at ground
- **Controlled source does NOT turn off!** $v_{be} = 0 - v_x = -v_x \neq 0$.
- v_x at emitter, R_L removed

CC Z_{out} — Simplified Circuit

Source removed $\Rightarrow$ base shorted to ground. v_x at emitter, R_L removed.



- **Controlled source does NOT turn off!** $v_{be} = -v_x \neq 0$ because v_x at E changes v_{be}.
- i_x flows back through r_π, $g_m v_{be}$, and R_E — all paths visible.

CC Z_{out} — Derivation

With v_x at the emitter and the base shorted to ground ($v_{be} = -v_x$):

The test voltage v_x pushes current back through r_π and the controlled source.

The impedance looking into the emitter (base grounded) is:

$$Z_{out,E} = \frac{r_\pi}{\beta + 1} \approx \frac{1}{g_m}$$

This is in parallel with R_E:

$$\boxed{Z_{out} = R_E \parallel \frac{r_\pi}{\beta + 1} \approx R_E \parallel \frac{1}{g_m}}$$

Impedance reflection in reverse!

Looking into the emitter, you see r_π **divided** by $(\beta + 1)$.

This makes Z_{out} very small.

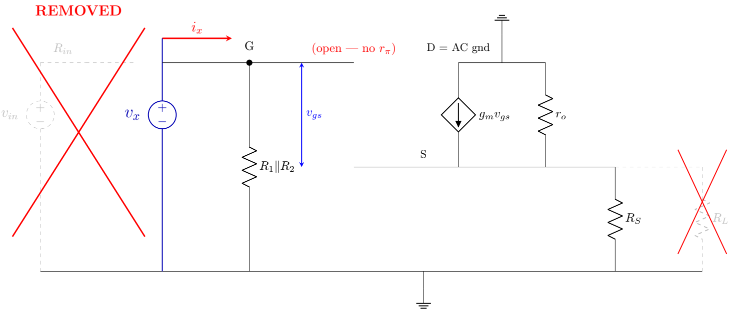
Numerical: $r_\pi/(\beta + 1) = 1636/101 = 16.2\ \Omega$

$$Z_{out} = R_E \| 16.2 = 1000 \| 16.2 = \boxed{15.9\ \Omega}$$

Razavi, Eq. 5.329 (with $R_S = 0$), p. 244

Finding Z_{in} for the CD Amplifier

CD Z_{in} Test Circuit



- Same as CS: gate is an **open circuit** — i_x only flows through $R_1 \| R_2$
- R_S at source does not affect Z_{in} (no current path to gate)

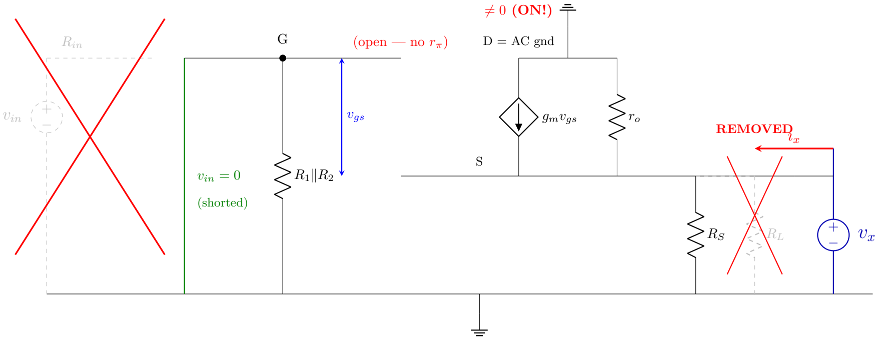
$$\boxed{Z_{in} = R_1 \| R_2}$$

Numerical: $Z_{in} = 400\text{ k} \| 100\text{ k} = \boxed{80\text{ k}\Omega}$ (same as CS)

Razavi, §7.4; gate input impedance is infinite (Ch. 7 summary, p. 340)

Finding Z_{out} for the CD Amplifier

CD Z_{out} Test Circuit



- Source removed (v_{in} and R_{in}) $\Rightarrow$ gate shorted to ground. v_x applied at source.
- **Controlled source does NOT turn off!** $v_{gs} = 0 - v_x = -v_x \neq 0$.
- $g_m v_{gs} = -g_m v_x$ acts like a conductance g_m from source to ground

CD Z_{out} — Derivation

With v_x at the source and gate grounded ($v_{gs} = -v_x$):
KCL at the source node (with $r_o = \infty$):

$$i_x = \frac{v_x}{R_S} + g_m v_x = v_x\left(\frac{1}{R_S} + g_m\right)$$

The controlled source contributes current $g_m v_x$, acting as a resistance $1/g_m$:

$$\boxed{Z_{out} = R_S \parallel \frac{1}{g_m}}$$

Numerical: $g_m = 3.12$ mA/V, $1/g_m = 320.5\ \Omega$

$$Z_{out} = 1000 \| 320.5 = \boxed{243\ \Omega}$$

CC/CD amplifiers have **low** Z_{out} — they are voltage **buffers**.
CC: 15.9 Ω CD: 243 Ω (vs. CE/CS: 1 kΩ)

Razavi, Eqs. 7.136–7.137, p. 334

All Four Topologies — Impedance Summary

Parameter	CE (BJT)	CS (MOSFET)	CC (BJT)	CD (MOSFET)
Z_{in}	$R_1 \| R_2 \| r_\pi$	$R_1 \| R_2$	$R_1 \| R_2 \| [r_\pi + (\beta+1) R_E]$	$R_1 \| R_2$
Z_{out}	$r_o \| R_C$	$r_o \| R_D$	$R_E \| \frac{r_\pi}{\beta+1} \approx R_E \| \frac{1}{g_m}$	$R_S \| \frac{1}{g_m}$
Z_{in}	650 Ω	80 kΩ	40.4 kΩ	80 kΩ
Z_{out}	1 kΩ	1 kΩ	15.9 Ω	243 Ω

CE/CS: High gain, but low Z_{in} (CE) and high Z_{out} — voltage amplifiers

CC/CD: Gain $\approx$ 1, but high Z_{in} and very low Z_{out} — voltage buffers

CC/CD Z_{out}: the controlled source **stays on**, unlike CE/CS where it turns off.

This active feedback is what creates the low output impedance.

CE/CS: Razavi Eqs. 5.224, 5.218 (Ch. 5); Eqs. 7.79, §7.2–7.3 (Ch. 7). CC/CD: Eqs. 5.326, 5.329 (Ch. 5); Eqs. 7.136–7.137 (Ch. 7).

The test source method works for ANY linear circuit

To find the impedance at any port:

1. Deactivate independent sources
2. Attach v_x, find i_x
3. $Z = v_x \,/\, i_x$

Reminder — the two-port gain formula:

$$\boxed{A_v = \frac{Z_{in}}{R_{in} + Z_{in}} \cdot A_{vo} \cdot \frac{R_L}{Z_{out} + R_L}}$$

Once you can find Z_{in}, Z_{out}, and A_{vo} for any amplifier, you can predict its gain with **any** source resistance and load.