EE461g: Introduction to Electronic Circuits

Lecture 21

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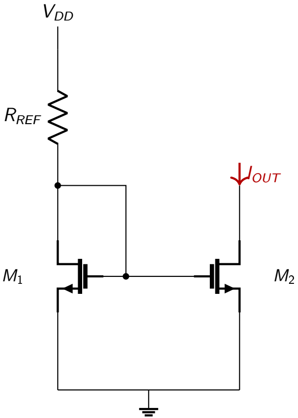
Outline

Cascode

Current Mirror

Review: Basic NMOS Current Mirror



From Lecture 17:

M_1 is diode-connected (gate = drain)

M_2 copies the current:
$I_{OUT} \approx I_{REF}$

Output resistance: $R_{out} = r_{o2}$

$r_o = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$

Problem: r_o is finite.
For $I_D = 450\ \mu\text{A}$, $\lambda = 0.02\ \text{V}^{-1}$:
$r_o = 111\ \text{k}\Omega$
The current changes if V_{DS} changes!

Why Do We Need Higher R_{out}?

In Lecture 20, the diff pair gain was $A_{diff} = g_m \left(r_{o2} \| r_{o4} \right)$

Higher r_o means higher gain

An ideal current source has $R_{out} = \infty$

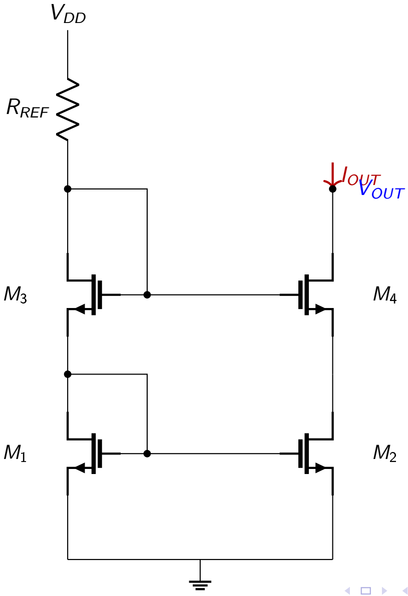
Goal: increase R_{out} of the current mirror without changing the bias current

Solution: Add a **cascode** transistor on top of the mirror transistor.
This multiplies the output resistance by $g_m r_o$.

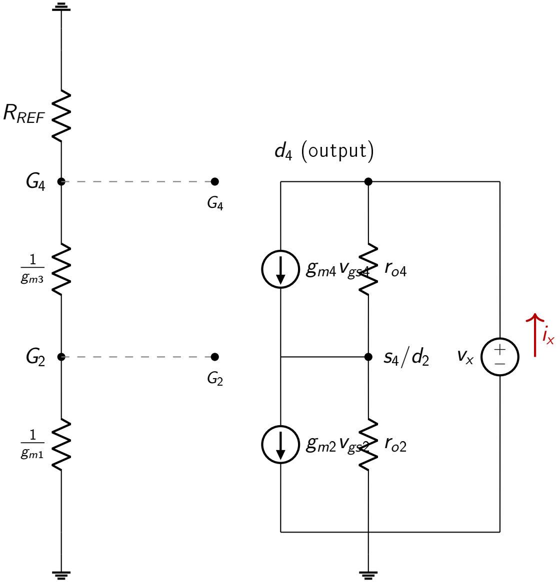
NMOS Cascode

Current Mirror

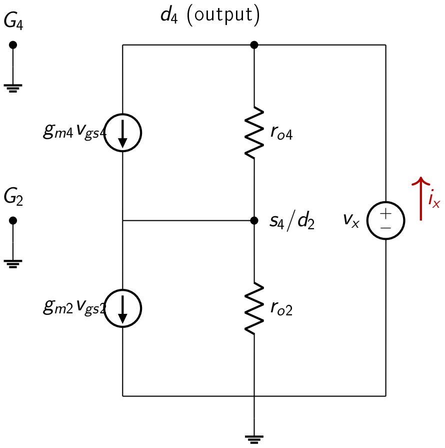
Cascode Current Mirror Circuit



Complete Small-Signal Model



Simplified Small-Signal Model for R_{out}



Simplifying the Dependent Sources

M_2: gate and source both at ground

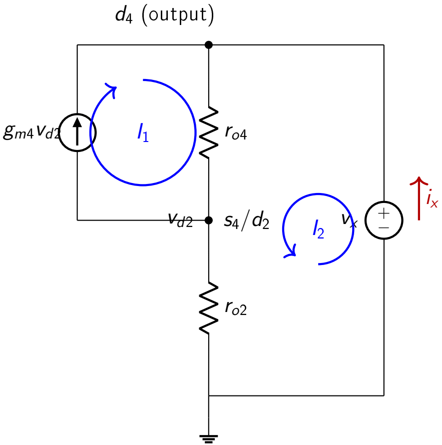
- G_2 is at ground (set by diode-connected M_1)
- S_2 is tied directly to ground
- Therefore $v_{gs2} = 0$ and $g_{m2}\, v_{gs2} = 0$
- M_2**'s current source is dead!** Only r_{o2} remains.

M_4: gate at ground, source at v_{D2}

- G_4 is at ground (set by diode-connected M_3)
- $S_4 = D_2 = v_{D2} > 0$ (when $v_x > 0$)
- Therefore $v_{gs4} = v_{G4} - v_{S4} = 0 - v_{D2} = -v_{D2}$
- The negative sign means current flows **opposite** to the arrow
- Equivalently: $v_{sg4} = +v_{D2}$, and we can **flip the arrow** to point upward

M_2 simplifies to just r_{o2}. M_4's dependent source becomes $g_{m4}\, v_{D2}$ with the arrow pointing **up** (from source toward drain).

Simplified Circuit for R_{out}



Deriving R_{out} from KVL

From KVL around the outer loop:

$$v_x = r_{o4}\left(i_x + g_{m4}\, v_{d2}\right) + r_{o2}\, i_x$$

We also know that:

$$v_{d2} = r_{o2}\, i_x$$

Substitute $v_{d2} = r_{o2}\, i_x$ into the first equation:

$$v_x = r_{o4}\left(i_x + g_{m4}\, r_{o2}\, i_x\right) + r_{o2}\, i_x$$

$$v_x = r_{o4}\, i_x \left(1 + g_{m4}\, r_{o2}\right) + r_{o2}\, i_x$$

$$v_x = i_x \left[r_{o4}(1 + g_{m4}\, r_{o2}) + r_{o2}\right]$$

$$\boxed{R_{out} = \frac{v_x}{i_x} = r_{o4} + r_{o2} + g_{m4}\, r_{o4}\, r_{o2} \approx g_{m4}\, r_{o4}\, r_{o2}}$$

The $g_{m4}\, r_{o4}\, r_{o2}$ term dominates because $g_m r_o \gg 1$.

Theory vs. SPICE: Output Impedance

	Formula	Theory	SPICE
NMOS ($I_D = 400\ \mu$A, $g_m = 1.3$ mA/V, $r_o = 125$ kΩ)			
Basic	r_o	125 kΩ	**132 kΩ**
Cascode	$g_m r_o^2$	19.8 MΩ	**20 MΩ**
Improvement	$g_m r_o$	158$\times$	152$\times$
NPN ($I_C = 400\ \mu$A, $g_m = 15.3$ mA/V, $r_o = 125$ kΩ)			
Basic	r_o	125 kΩ	**120 kΩ**
Cascode	$\beta\, r_o$	12.5 MΩ	**10 MΩ**
Improvement	β	100$\times$	83$\times$

NMOS cascode: theory matches SPICE almost exactly

BJT cascode: limited by β, not $g_m r_o$, because base current of Q_4 flows through the finite impedance at Q_3's collector

BJT cascode $R_{out} \approx \beta\, r_o$, not $g_m r_o^2$

Why the 5% Gap? A More Accurate r_o

MOSFET drain current in saturation:

$$I_D = \frac{K_P}{2}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

Small-signal output conductance (V_{GS} held constant):

$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \frac{K_P}{2}\frac{W}{L}(V_{GS} - V_{TH})^2 \cdot \lambda = \frac{\lambda I_D}{1 + \lambda V_{DS}}$$

Therefore:

$$r_o = \frac{1}{g_{ds}} = \frac{1 + \lambda V_{DS}}{\lambda I_D}$$

When $\lambda V_{DS} \ll 1$: $r_o \approx \frac{1}{\lambda I_D}$ (the equation sheet formula).

Example: $V_{DS} = 2.5$ V, $\lambda = 0.02$, $I_D = 400\ \mu$A:

$$r_o = \frac{1 + 0.05}{0.02 \times 400\mu} = \frac{1.05}{8\mu} = 131.3\ \text{k}\Omega \quad \text{(SPICE: 132 k}\Omega)$$

The simple $1/\lambda I_D$ gives 125 kΩ (5% low). Including $(1 + \lambda V_{DS})$ gives 131 kΩ — matching SPICE within 1%.

The Headroom Tradeoff

Downside of the cascode: it needs more voltage headroom.

Basic mirror: minimum $V_{OUT} = V_{OV}$ (one overdrive)

Cascode mirror: minimum $V_{OUT} = V_{OV2} + V_{GS4} = V_{OV2} + V_{TH} + V_{OV4}$

- Two overdrives + one threshold voltage
- With $V_{OV} = 0.2$ V and $V_{TH} = 0.5$ V: $V_{OUT,min} = 0.9$ V

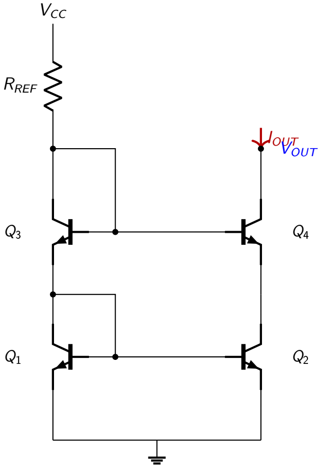
This reduces the output voltage swing

Tradeoff: Cascode gives much higher R_{out} (and therefore higher gain),
but requires more voltage headroom, reducing output swing.
Solution for low-voltage designs: **wide-swing** (folded) cascode.

BJT Cascode

Current Mirror

BJT Cascode Current Mirror



Same structure, NPN transistors:

Q_1/Q_2: basic mirror pair

Q_3/Q_4: cascode pair

Ideal: $R_{out} = g_m r_o^2$

But: base current of Q_4 limits R_{out} to $\approx \beta\, r_o$

Example: $I_C = 400\ \mu\text{A},\ \beta = 100,\ V_A = 50$ V:

$r_o = V_A/I_C = 125\ \text{k}\Omega$

$R_{out} \approx \beta\, r_o = 100 \times 125\text{k} = 12.5\ \text{M}\Omega$

SPICE: **10 MΩ**

BJT cascode limited by β, not $g_m r_o$. MOSFET cascode has no such limit ($R_{in,gate} = \infty$).

MOSFET vs. BJT Cascode Comparison

	NMOS	NPN
I_D (or I_C)	450 μA	450 μA
g_m	1.4 mA/V	17.3 mA/V
r_o	111 kΩ	111 kΩ
$g_m r_o$	156	1920
Basic R_{out}	111 kΩ	111 kΩ
Cascode R_{out}	17 GΩ	213 GΩ
Min V_{OUT}	$2V_{OV} + V_{TH}$	$2V_{CE,sat}$

Both MOSFET and BJT cascodes dramatically increase R_{out}.
BJT benefits more because $g_m r_o = V_A/V_T$ is inherently larger.

Lecture 21 Summary

Basic current mirror has $R_{out} = r_o$ (finite)

Adding a **cascode transistor** boosts R_{out} to $g_m r_o^2$

The cascode "shields" the mirror transistor from output voltage changes

Improvement factor: $g_m r_o$ ($\sim$100–1000$\times$)

Tradeoff: cascode needs more voltage headroom ($2V_{OV} + V_{TH}$)

Same technique works for both NMOS and NPN mirrors

Higher R_{out} in the mirror $\Rightarrow$ higher gain in the diff pair amplifier

Used extensively in op amp design