Consider the rectangular waveguide shown below.

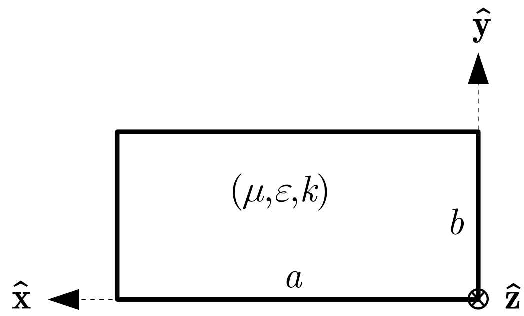

Figure 1: Rectangular waveguide geometry.

1 Magnetic Current Green's Function

1.1 Magnetic Field Due to a Magnetic Current

The magnetic field due to a Magnetic dipole $\mathbf{M} = \hat{\mathbf{l}}\delta(\mathbf{r} - \mathbf{r}')$ is given.

1.1.1 Mixed Potential Forumulation

The mixed-potential formulation for the magnetic field due to a magnetic current is

$$\mathbf{H}[\mathbf{M};\mathbf{r}] = -j\omega\mathbf{F}[\mathbf{M};\mathbf{r}] - \boldsymbol{\nabla}\Psi[\mathbf{M};\mathbf{r}] \tag{1}$$

where the electric vector potential is

$$\mathbf{F}[\mathbf{M};\mathbf{r}] = \iint_S \underline{\mathbf{G}}_F\left(\mathbf{r},\mathbf{r}'\right) \cdot \mathbf{M}(\mathbf{r}')dS' \tag{2}$$

and the magnetic sclar potential is

$$\begin{aligned} \Psi[\mathbf{M};\mathbf{r}] &= \iint_S q_m(\mathbf{r}')G_F\left(\mathbf{r},\mathbf{r}'\right)dS' & (3)\\ &= \frac{-1}{j\omega}\iint_S \left[\boldsymbol{\nabla}' \cdot \mathbf{M}(\mathbf{r}')\right] G_F(\left(\mathbf{r},\mathbf{r}'\right))dS' \ . & (4)\end{aligned}$$

Therefore,

$$\mathbf{H}[\mathbf{M};\mathbf{r}] = -j\omega\iint_S \underline{\mathbf{G}}_F\left(\mathbf{r},\mathbf{r}'\right) \cdot \mathbf{M}(\mathbf{r}')dS' + \frac{1}{j\omega}\boldsymbol{\nabla}\iint_S \left[\boldsymbol{\nabla}' \cdot \mathbf{M}(\mathbf{r}')\right] G_\Psi(\left(\mathbf{r},\mathbf{r}'\right))dS' \tag{5}$$

Here, the dyadic Green's function for the electric vector potential is

$$\underline{\mathbf{G}}_F\left(\mathbf{r},\mathbf{r}'\right) = F_x\left(\mathbf{r},\mathbf{r}'\right)\hat{\mathbf{x}} + F_y\left(\mathbf{r},\mathbf{r}'\right)\hat{\mathbf{y}} + F_z\left(\mathbf{r},\mathbf{r}'\right)\hat{\mathbf{z}} \tag{6}$$

and

$$F_x(\mathbf{r},\mathbf{r}') = \frac{\varepsilon}{j2ab}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\frac{\epsilon_{mn}}{\beta_{mn}}\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{n\pi}{a}x'\right)\cos\left(\frac{m\pi}{b}y\right)\cos\left(\frac{m\pi}{b}y'\right)e^{-j\beta_{mn}|z-z'|} \tag{7}$$

$$F_y(\mathbf{r},\mathbf{r}') = \frac{\varepsilon}{j2ab}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\epsilon_{mn}}{\beta_{mn}}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{n\pi}{a}x'\right)\sin\left(\frac{m\pi}{b}y\right)\sin\left(\frac{m\pi}{b}y'\right)e^{-j\beta_{mn}|z-z'|} \tag{8}$$

$$F_z(\mathbf{r},\mathbf{r}') = \frac{\varepsilon}{j2ab}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\epsilon_{mn}}{\beta_{mn}}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{n\pi}{a}x'\right)\cos\left(\frac{m\pi}{b}y\right)\cos\left(\frac{m\pi}{b}y'\right)e^{-j\beta_{mn}|z-z'|}\ . \tag{9}$$

The Green's function for the magnetic scalar potential is

$$G_\Psi(\mathbf{r},\mathbf{r}') = F_z(\mathbf{r},\mathbf{r}')\ . \tag{10}$$

In the above,

$$\epsilon_{mn} = \epsilon_m\epsilon_n, \tag{11}$$

where

$$\epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases} \tag{12}$$

and

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2} \qquad , \mathrm{Imag}\beta_{mn} \leq 0\ . \tag{13}$$