"Impulse Sampling"

Consider the cardinal series

$$\omega(t) = \sum_{n} \alpha_n 5a \left( \pi f_s (t - n/f_s) \right)$$

where

$$\alpha_n = \int_{-\infty}^{\infty} \omega(t) 5a \left( \pi f_s (t - n/f_s) \right) dt$$

$$f_s = \text{sampling rate (samples per second)}$$

$$T_s = \text{sampling period} = \frac{1}{f_s}$$

$$B = \text{bandwidth of } \omega(t), \text{ s.t.}$$

if $$\omega(f) = \text{figure}$$

then

$$\alpha_n = \omega \left( n/ f_s \right) = \omega \left( n T_s \right)$$

and $$f_s \geq 2B$$
It can be shown \( \phi_n(t) = 5a \left( \tau T_s (t - nT_s) \right) \) form an orthogonal set.

How do we implement or use this concept?

Impulse sampling

Assume \( g(t) = 0 \) for \(|t| > B\) i.e. \( g(t) \) is bandlimited.

\[ g_s(t) = g(t) \rho(t) = g(t) \sum_{n} \delta(t - nT_s) \rightarrow G_s(f) \]
We know the \( p(t) \) is
\[
\sum_{n} S(t-nT_s)^3 = \frac{1}{T_s} \sum_{k} S(t-kT_s)
\]

\( G_s(t) = G(t) \ast P(t) = G(t) \ast \frac{1}{T_s} \sum_{k} S(t-kT_s) \)

\( G_s(t) = \frac{1}{T_s} \sum_{k} G(t-kT_s) \)

Note: \( g_s(t) \) is still continuous since
\[
g_s(t) = g(t) \sum_{n} S(t-nT_s) = \sum_{n} g(t) \delta(t-nT_s) = \sum_{n} g(nT_s) \delta(t-nT_s)
\]
The discrete-time values are $g_d[n] = g(nT_s)$, so $g_s(t) = \sum_n g_d[n]S(t - nT_s)$.

Reconstruction

$C_{s}(t) = G_s(t) \ast P(t) \rightarrow H_r(t) \rightarrow G_r(t)$
\[ H_n(t) = T_s \text{rect} \left( \frac{f}{B_T} \right) \text{ where } 2B < B_T < 2 \left( \frac{1}{T_s} - B \right) \] 

Typically \( B_T = \frac{1}{T_s} \)

Practical configuration of Impulse Sampling

\[ g(t) \rightarrow x \rightarrow h_n(t) \rightarrow \hat{g}(t) \]

\[ \rho(t) = \sum_{n} s(t-nT_s) \]

Discrete-Time (DT) Summary

\[ g_s(t) = \sum_{n} g_d[n] s(t-nT_s) \]

\[ = \sum_{n} g_d[n] \delta(t-nT_s) \]
Analog to Digital Converters (A/D) or (ADC)

A/D converters are electronic devices which sample continuous time waveforms and output discrete sequences in the form of binary data.

Digital to Analog Converters (D/A) or (DAC)

DACs convert DT values to CT values.

Nyquist Sampling Rate

Sampling must be at a high enough rate else there will be distortion known as aliasing.
The Nyquist rate occurs when $B = \frac{1}{2T} - B$

$2B = \frac{i}{T} = f_{s, \text{min}}$

For 0 error in reconstruction, $f_s > 2B$

What does aliasing "sound" like?
Consider a monotonic audio tone

\[ g(t) = \cos 2\pi f_m t \quad \text{so} \quad \beta = f_m \]

Let's keep the sampling rate constant and vary \( f_m \) from a small value to an increasing one.
Anti-Aliasing Filters

In many cases, the signal being sampled has a wider bandwidth than allowed by the Nyquist rate, i.e. $B_{\text{max}} > \frac{1}{2T}$. Furthermore, it is practical to ignore the signal spectra greater than $\pm \frac{f_s}{2}$.

The implementation is $g(t) \rightarrow h_a(t) \rightarrow g_a(t)$.

Let $H_a(f) = \text{rect} \left( \frac{f}{2B_a} \right)$ where $B_a = \frac{1}{2T}$.

$C_a(f)$

Then the sampled $g_a(t)$ or $g_{as}(t)$ yields $C_{as}(f)$. 