Direct Sequence Spread Spectrum (DSSS)

The idea is to represent a "1" or "0" with a random function of sequence. These are called spread code functions or for this course "keys".

What is a "key"?

We are given a discrete-time channel with spectra $H_c[k]$. We incorporate the channel spectra into the design of the spread code.

For example

$$\tilde{z}_{\text{Key}}[n] = \tilde{\omega}_j[n] \ast h_c[n] \quad \text{for } n = 0, 1, \ldots, (N_{\text{key}} - 1)$$

where $h_c[n] \xrightarrow{\text{NFT}} H_c[k]$ and $\tilde{\omega}_j[n] \sim N(0, \frac{\sigma^2}{K_{\text{key}}})$
Consider a binary sequence where we have two spread codes for "1" and "0". The DSSS modulator is

\[ E \{ \tilde{\omega}_j[n] \}^3 = 0 \quad \forall j, n \]
\[ E \{ \tilde{\omega}_n^2[n] \} = \frac{\delta^2}{N_{\text{key}}} \]
\[ E \{ \tilde{\omega}_j[n] \tilde{\omega}_k[m] \} = 0 \quad \text{for } k \neq j \quad n \neq m \]

So we select \( \tilde{s}_{\text{key}, j}[n] \) sequences s.t.
\[ R_{j,k} = \sum_{n=0}^{N_{\text{key}}-1} \tilde{s}_{\text{key}, j}[n] \tilde{s}_{\text{key}, k}[n] \sim \delta \delta[t-j-k] \]
Bipolar detector for $M$ bit streams
Spread Code or Key Generation

Given $H_c[k]$ is the spectrum of the channel
where $h_c[n]$ be the impulse response
and $w_j[n]$ is a "white" noise sequence.

Form a large set of keys $M_{\text{key}}$ as

$$N_{\text{Sample}} \times 1 \begin{bmatrix} \forall j \in \text{key} \end{bmatrix} = \text{real} \left( \text{ifft} \left( \text{fft} \left( h_c \right), \ast \text{fft} \left( w_j \right) \right) \right)$$

which could be obtained by forming a large key
that is $M_{\text{key}} \times N_{\text{Sample}}$ long.

$$R_{\text{key}} = \begin{bmatrix} \begin{bmatrix} S^T_{\text{key,0}} & \vdots & S^T_{\text{key,m-1}} \end{bmatrix} \end{bmatrix}$$
Normalize \[ \sum_{k, n}^{T} \sum_{k, n}^{1} \]

The error is the absolute sum of the off diagonals.

\[
MSE = \frac{1}{(M_{xy})^2} \sum_{m=0}^{M} \sum_{n=0}^{M} (B_{xy}(m, n))^2 - (M_{xy}^{-1})_{\text{diagonal}}
\]

**Backward Elimination**

We systematically remove each key spread code from the set. Form the correlation matrix and determine MSE. This is done for all spread codes. The spread code to remove is associated with the highest MSE. Repeat this process until \( M_{\text{key}} \rightarrow M \)”