Predictive generic model control for non-linear interval systems with application in arc welding

Joseph Michael Istre and Yu Ming Zhang*

Department of Electrical and Computer Engineering,
University of Kentucky, Lexington, KY, USA
E-mail: jistre@lexmark.com    E-mail: ymzhang@engr.uky.edu
*Corresponding author

Abstract: A control technique is proposed for a class of non-linear systems with parameter intervals. This technique can be considered as a modification of the Generic Model Control (GMC), which specifies the desired derivative of the output as a Proportional-Integral (PI) function of the error between the reference and the output. Specifically, to reduce unnecessary control variations and improve the suitability for the control of non-minimum phase systems, a multistep prediction is incorporated into the GMC, which in its original form is ‘nearsighted’, to develop a Predictive GMC (PGMC). For non-linear interval systems, interval arithmetic is used to analyse the interval of the input, which can guarantee the system’s output stability defined by an output range. The PGMC is then derived for the interval non-linear system and an adaptive factor is proposed to improve the understanding of process dynamics and the performance of the closed-loop system. The resultant algorithm can control a non-linear system with parameter intervals and guarantee that the system’s output will meet a specified criterion. A simulation has been done to demonstrate the effectiveness of the PGMC in reducing control signal variations. Also, an arc welding process has been controlled using the interval PGMC algorithm to demonstrate the effectiveness of the full-proposed control technique.

Keywords: Generic Model Control (GMC); predictive control; arc welding; parameter interval; robust control.


Biographical notes: Joseph Michael Istre began researching the application of feedback control technologies to welding processes in 2003 while pursuing his Doctoral in Electrical Engineering at the University of Kentucky. He studied double-sided plasma arc welding and pulsed keyhole plasma arc welding under the professorship of Dr. Yu Ming Zhang. Particular research interests include non-linear process modeling, non-linear process control and machine vision. In addition, he has also received a Master of Mechanical Engineering and a Master of Business Administration at the University of Kentucky. Currently, he works as a Motion Control Engineer for Lexmark International.

Yu Ming Zhang is a Professor and the Director of Graduate Studies in the Department of Electrical and Computer Engineering at the University of Kentucky where he holds the James R. Boyd Professorship in Electrical Engineering and directs the Welding Research Laboratory and Applied Sensing and Control Laboratory. He received his PhD in Mechanical Engineering/Welding Major and MS and BS in Electrical Engineering/Control Major from the Harbin Institute of Technology, Harbin, China. He is a senior member of the IEEE and the SME and a member of the AWS and the ASME. He received the Donald Julius Groen Prize from the Institution of Mechanical Engineers, UK; the A.F. Davis Silver Medal award and the Adams Memorial Membership Award from the American Welding Society; and the 15th IFAC Triennial World Congress Poster Paper Prize and Application Paper Honorable Mention from the International Federation of Automatic Control.

1 Introduction

There is now a greater need for non-linear control to meet greater performance and cost demands in industry. Also, the capability to apply non-linear control techniques has risen due to the computational power increases and cost reductions of the controller hardware. Non-linear systems and their models are prevalent in industry; however, the true parameters of the model are often uncertain within a reasonable range and can change throughout time. Thus, a control algorithm that is easily implemented to control non-linear systems with parametric intervals is not only an interesting topic of research, but also could have extensive applications in the industrial setting.

This paper introduces an alteration to a non-linear feedback linearisation technique called Generic Model Control (GMC) by incorporating control predictions and parameter intervals. However, before outlining this new
algorithm, a short review of feedback linearisation, predictive control, and other popular non-linear control algorithms will be beneficial.

Feedback linearisation (Goodwin, 2001) is a conceptually simple technique for the control of non-linear systems with smooth non-linearities and stable process inversion. Consider the single-input–single-output non-linear state space system:

\[
\begin{align*}
\dot{x}(t) &= f(x) + g(x)u(t) \\
y(t) &= h(x)
\end{align*}
\]  

(1)

Assume that \( x = 0 \) is an equilibrium point of Equation (1), that is, \( f(0) = 0 \), and that the non-linear system has relative degree \( r \) defined in a certain neighbourhood \( U \) of \( x = 0 \). Apply a stable linear differential operator \( p(\rho) = p_0 \rho^r + p_1 \rho^{r-1} + \cdots + 1 \) of degree \( r \) to the system output \( y(t) \) such that

\[
p(\rho)y(t) = b(x) + a(x)u(t)
\]

(2)

where \( b(x) \) and \( a(x) \) are suitable non-linear functions of the state systems. Also, it is given that \( a(x) \neq 0 \forall x \in U \) as the non-linear system has relative degree \( r \) in \( U \). Hence, the control law below can be obtained as

\[
u(t) = \left[ p(\rho)y(t) - b(x) \right]/a(x)
\]

(3)

which is known as input–output feedback linearisation (Goodwin, 2001). One advantage of Feedback Linearisation is that it is simple and it allows a straightforward design of the differential operator \( p(\rho) \) as the roots of \( p(\rho) \) determine the dynamic behaviour of the output of the closed-loop system.

A completely different but well-known class of non-linear controllers that also directly use the non-linear model are Model Predictive Controllers (MPC) (Mayne et al., 2000). Linear MPC is a discrete time controller that calculates the present control, at each sampling time, by predicting over a horizon \( p \) the process response to changes in control. The change in control that is within specified constraints and that gave the most desirable process response is then implemented. Non-linear MPC (NMPC) is similar and is constructed as solving online finite horizon open-loop optimal control problem at each sampling time using the system model and predetermined state, input and output constraints. On the basis of the measurement samples obtained at time \( t \), the controller predicts the future dynamic behaviour of the system over a prediction horizon \( T_p \) and determines (over a control horizon \( T_c \leq T_p \)) the input such that a predetermined open-loop performance objective function is optimised. The disadvantages of NMPC are primarily due to the finite horizon optimal control problem possibly being non-convex. Non-convexity introduces the questions of how long will the optimisation take, will it terminate and is a suboptimal solution acceptable. Therefore, because of the non-convexity, NMPC formulations need to be derived that guarantee solution feasibility, robustness and performance despite the solution being suboptimal.

In addition to the feedback linearisation and model predictive control, there are also other commonly used non-linear control methods such as anti-windup (Kothare et al., 1994), backstepping (Krstic et al., 1995), \( H_\infty \) control, neural networks, fuzzy systems, etc. Specifically, anti-windup controllers are commonly used for situations involving actuator constraints or other similar limitations (Kothare et al., 1994). The anti-windup schemes involve mechanisms for notifying the controller when it is operating within a region that has certain constraints or limitations, and then the controller makes the predetermined modification to the control action. Backstepping is a method that can be used on non-linear systems of special structure to find an output having a passivity property, including the relative degree one and stable zero dynamics (Krstic et al., 1995). \( H_\infty \) can be viewed as a worst-case design methodology in the frequency domain where \( H_\infty \) stands for the space of complex functions, which are bounded and analytic in the closed right half of the complex plane (Maas, 1996). Neural networks are parameterised non-linear functions. Their parameters are, for instance, the weights and biases of the network. Adjustment of these parameters results in different shaped non-linearities. The neural network control is trained by adjusting the neural network parameters as a function of the error between the network output and a series of training data. Fuzzy controllers, which have the unique advantage of including the designer’s heuristics, can sometimes lead to better convergence properties for the actual input–output map. Fuzzy controllers are simply non-linear functions that are parameterised by, for example, the membership function and consequence parameters. For both neural network and fuzzy controller models, adaptive schemes have been studied and developed. Neural networks and fuzzy controllers are suitably designed to control non-linear systems (Abonyi et al., 1999; Babuska, 1998; Bossley, 1997; Botto et al., 1999; Clarke et al., 1989; Clarke and Mohtadi, 1989; Ferch et al., 1999).

Finally, GMC proposed by Lee and Sullivan (1988) is a model-based control method for linear and non-linear systems, which share certain similarities with the popular Proportional-Integral (PI) control. Instead of having the control as a PI function of \( y_{ref}(t) - y(t) \), where \( y_{ref} \) and \( y \) are the reference and system output, respectively; the GMC specifies a trajectory of \( y(t) \) as a PI function of \( y_{ref}(t) - y(t) \). Then the inverse process model is explicitly used to correlate \( y(t) \) with the input, and thus the GMC is model-based. In comparison with other model-based controls, the GMC does not directly use \( y_{ref} \) as the target for the output but rather specifies how the output should change.

The algorithm introduced in this paper expands the GMC algorithm to include predictive control and a process model with parameter intervals. The GMC algorithm was chosen to improve because it is model-based, simple and easy to use and has non-linear control capability.
2 Predictive generic model control

2.1 Generic model control

The most significant advantage of the GMC is that the transfer function of its closed-loop transfer function that is, from the reference to the output, can be designed without of the process model. Consider the process model that can be described by:

\[
\begin{align*}
\dot{y} &= a(x, u) \\
y &= f(x, u)
\end{align*}
\]

(4)

where \(y\) is the output and \(u\) is the input. If the desired system output is called, \(y_{ref}(t)\), then the GMC algorithm specifies a rate of change of the output, \(\dot{y}_{\text{Desired}}(t)\), with the following equation:

\[
\dot{y}_{\text{Desired}}(t) = k_1(y_{ref}(t) - y(t)) + k_2 \int_0^t (y_{ref}(\tau) - y(\tau))d\tau
\]

(5)

Then, the inverse process model is used to calculate the control action that makes the actual known output rate equal to the desired output rate (i.e. \(\dot{y} = \dot{y}_{\text{Desired}}\)) by solving the following equation for the control, \(u\):

\[
(\frac{\partial f(x, u)}{\partial x}) \dot{x} + (\frac{\partial f(x, u)}{\partial u}) \frac{du}{dt} = \dot{y}
\]

(6)

By applying the control action in Equation (6), the resultant closed-loop transfer function is

\[
\frac{Y(s)}{Y_{ref}(s)} = \frac{2\pi \xi s + 1}{\tau^2 s^2 + 2\pi \xi s + 1}
\]

(7)

where

\[
\tau = \frac{1}{\sqrt{k_2}}, \quad \xi = \frac{k_1}{2\sqrt{k_2}}
\]

(8)

Thus, a useful feature of the GMC algorithm is that, assuming no model uncertainties or disturbances, the closed-loop transfer function is only determined by (5) and is independent of the actual model. Hence, the GMC output trajectory can easily be designed through \(\dot{y}_{\text{Desired}}\) and \(k_1\) without using the process model and the process model is only used in the implementation phase.

Consider the discrete model

\[
\begin{align*}
x_{i+1} &= a(x_i, u_i) \\
y_{i+1} &= f(x_i, u_i)
\end{align*}
\]

(9)

Because \(y_i\) is the measured output and with GMC, we are trying to achieve \(\dot{y} = \dot{y}_{\text{Desired}}\), then the GMC rule in Equation (5) for digital systems can be written as

\[
\frac{y_{i+1,\text{Desired}} - y_i}{T_s} = k_1(y_{ref,\text{Desired}} - y_i) + (k_2T_s)
\]

(10)

Then, solving for \(y_{i+1,\text{Desired}}\) gives

\[
y_{i+1,\text{Desired}} = k_1T_s(y_{ref,\text{Desired}} - y_i) + (k_2T_s)y_i + \sum_{j=1}^{i-1}(y_{ref,j} - y_{k-j})
\]

(11)

where \(T_s\) is the sampling period. Then, for the class of systems that have an inverse model, the control \(u_i\) can be solved from the following control law:

\[
y_{i+1,\text{Desired}} = f(x_i, u_i)
\]

(12)

For convenience, the inverse of the above equation from which the GMC is calculated will be denoted as

\[
u_i = u_{\text{GMC}}(x_i, y_{i+1,\text{Desired}})
\]

(13)

The discrete GMC algorithm is thus:

\[
u_i = u_{\text{GMC}} \left( \sum_{j=1}^{i-1}(y_{ref,j} - y_{k-j}) + y_k \right)
\]

(14)

where the original GMC parameters \(k_1\) and \(k_2\) are still kept in the algorithm because of their direct relationship with the closed-loop system’s transfer function.

2.2 Predictive generic model control

When disturbances and noise exist or the dynamics of the process are not accurately modelled, the proportional part of the GMC calculation \(k_1(y_{ref}(t) - y(t))\) when projected through the inverse process model can cause intense control fluctuations. Thus, a direct use of GMC can produce a control signal that frequently oscillates, which cannot be implemented in real hardware. A process constraint upon the change in the control could be given, but the calculation of the GMC with process constraints becomes more complex requiring non-linear programming optimisation (Ansari and Tade, 2000). Also, an output pre-filter could be used to reduce the output noise that produces the controller variation. However, a pre-filter’s effect on the disturbance and model inaccuracy/mismatch is limited. For example, the nominal model used may have a dominant time constant, which is much smaller than the actual one. This would tend to result in a faster control, which can generate oscillation in the output and the pre-filter will be ineffective in overcoming the effect of this model inaccuracy/mismatch.

The Predictive GMC (PGMC) proposed in this paper reduces the control signal variation associated with the original GMC by averaging predicted future controls that are each calculated using the original GMC method. To illustrate, PGMC uses the original GMC method and the non-linear process model in a ‘for’ loop to calculate totally \(p\) predicted controls and outputs as:

\[
u_0, \hat{G}, \hat{G}^{-1}, \hat{G}_{k+1}, \hat{G}_{k+2}, \ldots, \hat{G}_{k+p}
\]

(15)

where \(\hat{G}\) and \(\hat{G}^{-1}\) denote the nominal model and its inverse that is used for the control calculation. Mathematically, to implement the PGMC, the following recursive equations are calculated for \((i = 0, 1, 2, \ldots, p-1)\):
Predictive generic model control

\[ \hat{u}_{k+i} = u_{\text{GMC}} \left( \sum_{j=0}^{k} \left( y_{\text{ref},k+i-j} - \hat{y}_{k+i-j} \right) + (k_i T_i) T_i \right) \]  

(16)

\[ \hat{y}_{k+i} = x(\hat{x}_{k,i}, \hat{u}_{k+i}) \]  

(17)

where

\[ \alpha_j = \frac{p - j}{(1 + 2 + \cdots + p)}, \quad j = \{0, 1, \ldots, p-1\} \]  

(19)

The actual control implemented, \( u_c \), is set equal to a weighted average of the predicted controls:

\[ u_c = \alpha_0 \hat{u}_i + \alpha_1 \hat{u}_{k+1} + \cdots + \alpha_{p-1} \hat{u}_{k+p-1} \sum_{j=0}^{k-p+1} \alpha_j \hat{u}_j \]  

(18)

3 Predictive generic model control of interval models

In many applications, the exact parameters of the model are uncertain and can change throughout time within a known interval. Thus, the use of the parameter interval in the control law is desirable, and the next sections of this paper incorporate the parameter interval to the PGMC algorithm to achieve a more robust control of a model with parameter uncertainties.

3.1 Interval mathematics review

A real scalar interval \([Y] \in R\) is a closed, connected and bounded subset of \(R\), such that

\[ [Y] = [Y^- Y^+] = \{Y \mid Y^- \leq Y \leq Y^+\} \]  

(21)

Interval arithmetic is generalised for addition, subtraction, multiplication and division (Hansen and Walster, 2004).

Given the scalar interval mathematics problem as follows:

\[ [Y^- Y^+] = [B^- B^+] \times [U^- U^+] + [H^- H^+] \]  

(22)

where the intervals \([B], [U]\) and \([H]\) are known. The interval \([Y]\) can be found using:

\[ Y^- = \min \left( B^- U^- + B^- U^+ + B^+ U^- + B^+ U^+ \right) + H^- \]  

\[ Y^+ = \max \left( B^- U^- + B^- U^+ + B^+ U^- + B^+ U^+ \right) + H^+ \]  

(23)

If given an inverse interval problem where the goal is to find the \([U]\) that satisfies a particular \([Y]\), then two assumptions will be made to obtain an analytic solution:

1. \([B] > 0\) \((B^+, B^- > 0)\)
2. the given interval \([Y]\) is wider than the given interval \([H]\).

The solution to this linear inverse interval problem can only be solved in general through non-linear programming methods (Hansen and Walster, 2004). However, under these two assumptions, it easily shown that (Istre, 2004):

\[ U^- = \max \left( \frac{Y^- - H^-}{B^-}, \frac{Y^- - H^+}{B^+} \right) \]  

\[ U^+ = \min \left( \frac{Y^+ - H^+}{B^-}, \frac{Y^+ - H^-}{B^+} \right) \]  

(24)

This calculation for an inverse interval \([U]\) that satisfies a given output criterion \([Y]\) is an interesting calculation because this can be used to bound a digital control at each sampling time to ensure a certain output criterion, assuming the two conditions used to give the solution in (24) are satisfied.

Thus, the inverse interval problem can be used to calculate an acceptable range of \(U\), called \([U]\), that satisfies a stability criterion defined by \([Y]\) where the ‘S’ subscript signifies that the interval is used for defining a ‘stability’ interval. Then for the output criterion \([Y]\) = \([Y_{\min} Y_{\max}\] , the acceptable \([U]\) would be found using the following equation:

\[ [U] = \max \left( \frac{Y_{\min} - H^-}{B^-}, \frac{Y_{\min} - H^+}{B^+} \right), \min \left( \frac{Y_{\max} - H^-}{B^-}, \frac{Y_{\max} - H^+}{B^+} \right) \]  

(25)
The extension to dynamical systems of the stability interval $[U_i]$ that maintains a stability output interval, will be made more clear in the next sections.

### 3.2 Non-linear interval model

Consider a class of non-linear scalar models:

$$
\begin{align*}
    x_{i+1} &= f(x_i, u_k) \\
    y_{i+1} &= \sum_{i=1}^{n} \varphi_i f_i(x_i) + \beta g(x_i) u_k
\end{align*}
$$

(26)

where $x$ is the state vector of the non-linear system, $f_i$'s and $g$ are non-linear scalar functions of the state and $\varphi_i$'s and $\beta$ are parameters bounded as intervals $\varphi_i^+ \leq \varphi_i \leq \varphi_i^-$ and $0 < \beta^- \leq \beta \leq \beta^+$ and $g(x) > 0$. It is apparent that the model is linear with respect to the parameters of the model and is a subcategory of the non-linear process defined by Equation (9).

Assume that the state $x$ is fully known at each time $k$ such that $f_i(x_i), f_i(x_i), ..., f_i(x_k)$ and $g(x_i)$ are also known. Then, the output equation with parameter intervals can be reformulated at each sampling time into an interval equation:

$$
\begin{align*}
    [Y_{i+1} Y_i^+] &= \sum_{i=1}^{n} \varphi_i f_i(x_i) \\
    &+ [\beta^+ \beta^-] g(x_i) [U_i^+ U_i^-] \\
\end{align*}
$$

(27)

Or it can be more clearly organised into the form of Equation (22) to:

$$
\begin{align*}
    [Y_{i+1} Y_i^+] &= \left[ H_i^+ H_i^- \right] + \left[ B_i^+ B_i^- \right] [U_i^+ U_i^-] \\
\end{align*}
$$

(28)

where

$$
\begin{align*}
    H_i^+ &= \sum_{i=1}^{n} \min \left( \varphi_i^+ f_i(x_i), \varphi_i^- f_i(x_i) \right) \\
    H_i^- &= \sum_{i=1}^{n} \max \left( \varphi_i^+ f_i(x_i), \varphi_i^- f_i(x_i) \right) \\
    B_i^+ &= \beta^+ g(x_i) \\
    B_i^- &= \beta^- g(x_i)
\end{align*}
$$

(29)

The utility of (27) is that every sampling time the non-linear model is reduced to an interval equation of the form given in Equation (28). If $[B_i] > 0$ and $[Y_i]$ are wider than $[H_i]$, then there are a closed-form solution for the inverse interval problem and Equation (25) can be used to find a $[Y_i]$ that satisfies a given $[Y_i]$ criterion. Then, at each sampling time, the actual control that it used, $u_i$, should, prior to implementation, be bounded by $[U_i]$ to ensure the output criterion of $[Y_i]$, as in the following equations:

$$
\begin{align*}
    [U_i] &= \left[ \begin{array}{c}
        \max \left( Y_{i,\text{max}} - H_i^+, y_i_{\text{min}} - H_i^- \right) \\
        \min \left( Y_{i,\text{min}} - H_i^-, y_i_{\text{max}} - H_i^+ \right)
    \end{array} \right] \\
\end{align*}
$$

(30)

Then, the actual control that is implemented is $\bar{u}_i$.

### 3.3 Generic model control

For the system in Equation (27), GMC is a function of the intervals $[H_i]$ and $[B_i]$. Assuming that it desirable to centre the desired output, $y_{\text{desired},i+1}$, in the output interval, $[Y_i]$, it is easily shown that the GMC to accomplish this would be to use the average of the $[H_i]$ and $[B_i]$ interval endpoints so that:

$$
\begin{align*}
    u_i &= \frac{y_{\text{desired},i+1} - (H_i^++H_i^-)/2}{(B_i^++B_i^-)/2} \\
    &= U_i \text{GMC} \left( \bar{H}_i, \bar{B}_i, y_{\text{desired},i+1} \right)
\end{align*}
$$

(32)

where

$$
\begin{align*}
    \bar{H}_i &= (H_i^++H_i^-)/2 \\
    \bar{B}_i &= (B_i^++B_i^-)/2
\end{align*}
$$

(33)

and

$$
\begin{align*}
    y_{\text{desired},i+1} &= T_i \left( k_i \left( y_{\text{ref},i} - y_i \right) + K_i \sum_{j=i}^{k_i} \left( y_{\text{ref},j} - y_j \right) \right) + y_i
\end{align*}
$$

and $T_i$ is the sampling period.

### 3.4 Interval generic model control

Now in Equation (27), essentially there are two models, a maximum and minimum model, which are more clearly defined in Equations (28) and (29). However, if the real process is operating more closely to the maximum model, it would be advantageous to put greater emphasis on the inverse maximum model that is used in each of the control calculations. On the other hand, if the real process is operating more closely to the minimum model, it would be advantageous to put greater emphasis on the inverse minimum model that is used in each of the control calculations. A simple approach is to use a ratio, $F_i$, of the two models, where the ratio is found from the previous output sample and what the maximum and minimum models predicted for that time. However, for some processes this ratio will change too quickly due to noise. Thus, the ratio must be filtered to match the likely movement of the process throughout its maximum and minimum models. Finally, this max–min combination model can be used to calculate the next control, but before it is implemented the control, $u_i$, must be constrained to the bounds of $[U_i]$ to ensure the output criterion of $[Y_i]$. This is the idea that is proposed more mathematically here.

In general, the actual output may not locate at either the maximum model’s output or the minimum model’s output but somewhere in the middle as in Figure 1.
For each sampling time, a prediction of the likely process’s output in relation to the maximum and minimum models’ outputs can be made based on what the relation has been in the past. Thus, if this relation is defined using, \( F_k \), called the adaptive factor, by:

\[
y_k = Y_k^* + F_k \left( Y_k^* - Y_k^- \right)
\]

(34)

where \( Y_k^* \) and \( Y_k^- \) are the minimum and maximum models’ predicted outputs for time \( k \). Then, the next sampling time this adaptive factor would be defined as

\[
F_{k+1} = \left\{ \frac{Y_{k+1}^* - Y_{k+1}^-}{Y_{k+1}^* - Y_{k+1}^-} \right\}
\]

(35)

However, the real value of \( F_{k+1} \) is unknown so it can be made equal to an average of its previous history \( F_k, F_{k-1}, \ldots, F_0 \) or made closer to 1 or 0 to emphasise either the maximum or minimum model, respectively. At each sampling time, once \( F_{k+1} \) is chosen by the algorithm, the GMC is no longer calculated using Equation (32) but instead by using Equation (36). The definitions of \( [H_k], [B_k] \) are given in Equation (29) and \( Y_{k+1,\text{Desired}} \) is given in Equation (11).

\[
u_k,\text{GMC} = \frac{Y_{k+1,\text{Desired}} - \left( H_{k+1}^- + \hat{F}_{k+1} \left( H_{k+1}^* - H_{k+1}^- \right) \right)}{B_{k+1}^- + \hat{F}_{k+1} \left( B_{k+1}^* - B_{k+1}^- \right)}
\]

(36)

For the process in this paper, the adaptive factor does not change very rapidly. Thus, \( F_k \), was estimated with satisfactory accuracy simply using the average of \( F_k, (j = 0,1,\ldots,m-1) \) resulting in

\[
\hat{F}_{k+1} = \frac{1}{m} \sum_{j=0}^{m} F_{k-j}
\]

(37)

After the control is calculated using Equation (36), the control must also be constrained by \([U_j]\), using Equations (30) and (31) to find the control, \( \hat{u}_k \), that ensures that the output criterion \( [Y_j]\) is satisfied.

### 3.5 Interval predictive generic model control

Now the algorithm described in Section 3.4 will be expanded to include control predictions and will result in the final algorithm for which this paper is written. Interval PGMC is a true combination of Sections 2 and 3. Interval PGMC has a desired output trajectory according to the GMC algorithm, Equation (5), incorporates the adaptive factor estimation, Equation (34), calculates \( u_k,\text{GMC} \) by Equation (36), and then constrains the average of the predicted controls, Equation (18), to be within the stable control interval \([U_j]\), Equation (25).

This can be restated mathematically by generalising the interval control in Equations (36)–(38) so that it can be used for control predictions. Also, like in PGMC, the future controls were averaged so it is with interval PGMC as given by Equation (40). Finally, to ensure that the output criterion is met, the averaged control is constrained according to Equation (41).

\[
u_k,\text{PGMC} = \min \left[ \left( U_{k+1,\text{PGMC}}^* \right), \max \left( U_{k+1,\text{PGMC}}^* \right) \right]
\]

(40)

To sum up more clearly, essentially the steps of the algorithm are as follows:

1. sample the process, \( y_k \)
2. estimate \( \hat{F}_{k+1} \) using either previous measured \( F_k \)'s or forcing it closer to 0 or 1

Figure 1: Depiction of process output in relation to the maximum and minimum models’ outputs.
3 calculate \( u_{t,\text{GMC}} \) using Equation (38) using the max and min models, \( \hat{F}_{t+1} \)

4 find a predicted output \( \hat{y}_{t+1} \).

5 repeat the cycle of steps 1–4, \( p \) times

6 then find an average of the predicted controls:

\[
u_{t,\text{PGMC}} = \frac{1}{p} \sum_{i=t}^{t+p-1} \alpha_i \hat{u}_{i,\text{GMC}}\]

7 use Equation (41) to constrain \( u_{t,\text{PGMC}} \) by the stability interval \([U_k]\) to calculate \( \hat{u}_t \).

8 implement \( \hat{u}_t \).

It should be noted that all the information is already available for another possibility in simulating the future \( p \) controls. The alternative would be to constrain each of the future controls by their respective simulated future stability intervals at each iteration of the \( p \) prediction process. This would either make the control signal even smoother or unnecessarily restrict the control signal. For the welding experiments in Section 4.2 only the final control was constrained as in step 7 above.

Finally, a further theoretical comment regarding the use of parameter intervals will be useful before presenting the experimental results of this algorithm in the next section. It is not always simple to obtain the exact intervals for each parameter of the process. However, it is more straightforward to find a likely distribution of each parameter. Then, assuming a normal distribution for each parameter, the parameter interval can be selected based upon the 1 sigma, 3 sigma or 6 sigma, etc. values of the distribution with a corresponding statistical percent confidence that the real process will behave according to that model. This is a useful tool to make the control algorithm more aggressive by narrowing the parameter intervals and to estimate how often the output criterion will not be satisfied.

4 Examples

4.1 PGMC simulation

In a review, GMC is a feedback linearisation technique for non-linear systems. PGMC has the advantage of reducing GMC’s controller variation and interval PGMC has the further advantage of incorporating parameter intervals in the control algorithm to increase robustness. Non-interval PGMC will be demonstrated with two linear system simulations to show the reduction in the control signal variation compared with the regular GMC algorithm. Then, the interval PGMC method will be demonstrated in a practical application using a pulsed keyhole plasma arc welding process.

To demonstrate the advantages associated with PGMC in comparison with the original GMC method, an underdamped linear system and an overdamped linear system are selected to simulate the closed-loop performance. The underdamped system is given by

\[
G(s) = \frac{200}{s^2 + 9s + 200} \quad \text{or} \quad G(z) = \frac{0.02}{1.11z^2 - 2.09z + 1} \quad \text{for} \quad T_s = 0.01 \text{sec}
\]

Its open-loop settling time is 0.8 sec and the open-loop gain is 1. It is desired to reduce the settling time by \( \frac{1}{2} \). Thus, the closed-loop system’s \( \xi \) and \( \tau \) in (7) are set as \( \xi = 4 \) and \( \tau = \frac{0.4}{0.43} \) and the number of the prediction steps \( p \) is set at 14, based on the ‘rule of thumb’ PGMC gain design given in Equation (20).

The closed-loop output and control for both GMC and PGMC are plotted in Figure 2. As can be seen, the PGMC achieved a 75% reduction in the control variation, \( \sum_{t=1}^{N} |u_t - \hat{u}_t| \), with respect to the GMC. The actual PGMC settling time of the closed-loop system equals 0.33 sec, which is slightly faster than the 0.4 specified by ‘rule of thumb’ design because of the underdamped-open loop dynamics.

The overdamped system is

\[
G(s) = \frac{1.5}{s^2 + 14s + 40.02} \quad \text{or} \quad G(z) = \frac{0.00015}{1.44z^2 - 2.14z + 1} \quad \text{for} \quad T_s = 0.01 \text{sec}
\]

Its open-loop settling time is 1.16 sec and the open-loop gain is equal to 0.0375. It is desired to reduce the settling time by \( \frac{4}{5} \). The parameters for the PGMC are \( \xi = 4 \), \( \tau = 0.232/0.43 \) sec and \( p = 7 \). The closed-loop output and control for both GMC and PGMC are plotted in Figure 3.
It can be seen that the PGMC achieved a 69% reduction in the control variation with respect to the GMC. The actual PGMC settling time of the closed-loop system equals 0.275 sec, which is slightly slower than the 0.232 specified by the ‘rule of thumb’ design due to the overdamped open-loop dynamics. Nonetheless, the important point is that the controller variation for both of these examples is significantly reduced by averaging predicted controls without significantly altering the system output behaviour.

4.2 Experimental result of interval PGMC

Pulsed keyhole plasma arc welding is a novel welding process developed at the University of Kentucky (Lu et al., 2004).

As shown in Figure 4, a peak current is applied until a keyhole is sensed by the backside sensor, and then the current is switched to a much lower base current to let the keyhole close. The backside sensor senses a quick voltage change when the keyhole begins to open due to escape of the plasma charge. The keyhole is defined to be ‘open’ when the voltage exceeds 1/2 volt. The goal is to be able to apply just the right peak current so that the time it takes to establish the keyhole is equal to a specified time. The phenomenological model structure for this process was developed by a consideration of the energy input into the system and is given as:

\[ T_{p,k} = a_0 + a_1 I_{p,k-1} + a_2 I_{p,k-2} T_{p,k-1} + a_3 I_{p,k-2} T_{p,k-2} \]  

where the input of the process \( u = I_p \) is the amperage of the peak current which is applied to establish the keyhole (a cavity in the molten metal through the thickness of the work-piece), and the output of the process \( y = T_p \) is the period of the peak current applied. The objective is to have the keyhole established in a given period \( y_{\text{ref}} = T_p \) every cycle by adjusting the amperage of the peak current \( u = I_p \).

For this process, system identification experiments were completed using random inputs within the possible process operating range, and a least squares algorithm was employed for determining the process parameters. This was completed four times to enable the construction of the probable max and min of each of the four model parameters. The range of each parameter is given as follows:

\[ a_{0,\text{max}} = 734.66, \quad a_{0,\text{min}} = -4.96, \]
\[ a_{2,\text{max}} = 0.7 \times 10^{-3}, \quad a_{2,\text{min}} = -14 \times 10^{-4} \]
\[ a_{0,\text{min}} = 707.15, \quad a_{0,\text{min}} = -4.53, \]
\[ a_{2,\text{min}} = -1.3 \times 10^{-1}, \quad a_{3,\text{min}} = -4 \times 10^{-4} \]

In using the interval PGMC methodology with the adaptive factor

\[ \hat{F}_{k+1} = \frac{1}{m} \sum_{j=0}^{m-1} F_{k-j} \]

the tuneable control parameters and output reference and output criteria were selected as follows:

\[ T_s = 1 \text{ cycle}, \quad T_{\text{sett,desired}} = 3 \text{ cycles}, \]
\[ p = 2 \text{ cycles}, \quad m = 4 \text{ cycles}, \]
\[ T_{p ob} = 300 \text{ ms}, \quad T_{p,\text{min}} = 0 \text{ ms}, \quad T_{p,\text{max}} = 1000 \text{ ms} \]

Thus, for the pulsed keyhole plasma arc welding process the goal is to find, within a few cycles, the proper peak current that establishes the keyhole for \( T_p = 0.3 \) sec. The output criteria of \( T_{p,\text{min}} \) and \( T_{p,\text{max}} \) specify, through the max and min models, a peak current interval, \( [I_{p,max}, I_{p,min}] \), that constrains the calculated control peak current, \( I_{p,k} \).

The results of two different closed-loop control experiments are illustrated in Figures 5 and 6. Four graphs are shown for each figure – the peak current versus cycle number, the time to establish the keyhole with a reference of 0.3 sec versus cycle number, the value of the adaptive factor versus cycle number and finally a histogram of the times to establish a keyhole.
The first thing to note in the graphs of the peak time versus cycle number in both Figures 5 and 6, is that the output has reached a quasi-steady state centred about the reference within 5–10 cycles. This is slightly longer than specified by the control parameters; however for this process, the first few cycles are used just to heat the workpiece. At the beginning of the experiment, the peak time varied significantly until the peak current reached a proper value. Moreover, in Figure 5, one can see that although the output has reached a quasi-steady state, the peak current is actually still slowly increasing to achieve the amount of energy input necessary to maintain the desired peak time.

Also, it is interesting to note three attributes of the movement of the adaptive factor in both Figures 5 and 6. Firstly, it varies significantly at the beginning of the process while the control signal is also changing quickly. However, as the process output has reached steady state, though the adaptive factor is changing significantly, the control signal is not changing significantly. This is because as the process reaches steady state, the max and min models for this process begin to converge and the differences in the control calculated from each decreases. Finally, during the entire experiment, the adaptive factor is closer to 0 than to 1, which suggests that the minimum model is more accurately describing the process. In fact, when the adaptive factor reaches 0, this suggests that the minimum parameters used in the control calculations are likely not the actual minimum parameters.

However, the important point is that when the minimum parameters used to constrain the control were not the real minimum process parameters, the control still achieved the reference and the system did not become unstable. The closed-loop system will not be able to guarantee the output criteria with 100% confidence, but the output failure rate can be quantified based upon the estimated distribution of the parameters and the known model structure. Thus, as with this example, a smaller interval of the parameters can be used to obtain higher closed-loop performance with an understanding of the statistical failure rate.

The last graph in both Figures 5 and 6 is the histogram of the peak times and demonstrates the effectiveness of the closed-loop system’s ability, with disturbances, to meet the output reference.

Figures 7 and 8 show the welds made. The physical goal of the pulsed keyhole plasma process is to obtain slightly overlapping keyhole penetration spots. This can be particularly seen in the zoomed backside view of the weld in the image shown in Figure 7. The goal of these overlapping penetration spots is to minimise the energy input that will achieve a quality weld.

The first thing to note in the graphs of the peak time versus cycle number in both Figures 5 and 6, is that the output has reached a quasi-steady state centred about the reference within 5–10 cycles. This is slightly longer than specified by the control parameters; however for this process, the first few cycles are used just to heat the workpiece. At the beginning of the experiment, the peak time varied significantly until the peak current reached a proper value. Moreover, in Figure 5, one can see that although the output has reached a quasi-steady state, the peak current is actually still slowly increasing to achieve the amount of energy input necessary to maintain the desired peak time.

Also, it is interesting to note three attributes of the movement of the adaptive factor in both Figures 5 and 6. Firstly, it varies significantly at the beginning of the process while the control signal is also changing quickly. However, as the process output has reached steady state, though the adaptive factor is changing significantly, the control signal is not changing significantly. This is because as the process reaches steady state, the max and min models for this process begin to converge and the differences in the control calculated from each decreases. Finally, during the entire experiment, the adaptive factor is closer to 0 than to 1, which suggests that the minimum model is more accurately describing the process. In fact, when the adaptive factor reaches 0, this suggests that the minimum parameters used in the control calculations are likely not the actual minimum parameters.

However, the important point is that when the minimum parameters used to constrain the control were not the real minimum process parameters, the control still achieved the reference and the system did not become unstable. The closed-loop system will not be able to guarantee the output criteria with 100% confidence, but the output failure rate can be quantified based upon the estimated distribution of the parameters and the known model structure. Thus, as with this example, a smaller interval of the parameters can be used to obtain higher closed-loop performance with an understanding of the statistical failure rate.

The last graph in both Figures 5 and 6 is the histogram of the peak times and demonstrates the effectiveness of the closed-loop system’s ability, with disturbances, to meet the output reference.

Figures 7 and 8 show the welds made. The physical goal of the pulsed keyhole plasma process is to obtain slightly overlapping keyhole penetration spots. This can be particularly seen in the zoomed backside view of the weld in the image shown in Figure 7. The goal of these overlapping penetration spots is to minimise the energy input that will achieve a quality weld.
Predictive generic model control

4.3 Experimental results of interval PGMC with tighter output constraints

The pulsed keyhole plasma process is again used here for the implementation of the non-linear interval PGMC but with a tighter output criterion so that the stabilising controls interval will affect the control signal. The experimental results are shown in Figures 9–11.

For the third experiment, the parameter intervals that were found from system identification were used in the process control, and the closed-loop results achieve what is expected. In addition, the reference peak time was changed midway during the experiment to view the closed-loop behaviour from one steady state to another. However, for the fourth and fifth set of experiments the parameter intervals were significantly narrowed about each of their mean with the expectation that the process would likely operate outside of the process intervals. Also, the adaptive factor was constrained to remain between 0 and 1 for experiment 4 and between 0 and 1.2 for experiment 5 to measure the process's movement out of the max and min models. The results of three different closed-loop control experiments are illustrated in Figures 9–11. To enhance understanding of the process, the graphs show slightly different information than the previous Figures 5 and 6. Four graphs are shown for each figure – the keyhole voltage measurement versus time, the plasma current versus time, the time to establish the keyhole and its reference versus time and the value of the adaptive factor versus time. Images of the welds are shown in Figure 12.

As stated before, Figure 9 demonstrates the algorithm with the parameter intervals found from system identification, whereas Figures 10 and 11 use tighter intervals. In Figure 9, the output performs satisfactorily, and one can see the control compensate to make the system output adjust to the second reference point. However, in Figure 10, as the process operated out the max and min models used in the control calculation, then the control could not properly adjust to achieve the desired closed-loop reference. In Figure 11, one can see how much the process operated outside of the max and min models as the restriction on the adaptive factor to be between 0 and 1 was removed. An interesting conclusion of these experiments is that movement of the adaptive factor can show the control designer something about the accuracy of the parameters intervals. Moreover, to put a greater emphasis on either the max or min model in the control calculation, the control designer can put restrictions on the movement of the adaptive factor to be either at or near one or at or near zero.

Finally, in designing a controller with this algorithm, in addition to the selection of the GMC gains and number of prediction steps, particular consideration must be given in the selection of the parameter intervals, adaptive factor filter and output criterion. With overly wide parameter intervals, the control may not be aggressive enough. However, with overly narrow parameter intervals the control will, of course, not guarantee the output criterion but more importantly may unable to achieve the desired closed-loop performance, as in Figure 10. Also, if the max and min models require a substantial difference in control to achieve the desired output, then the adaptive factor filter directly influences the control variation. Finally, it is important that the output criteria is selected so that there is always a solution to the interval problem, namely that $\{Y_h\}$ is a larger interval than $\{H_h\}$, as in the assumptions for Equation (24).
5 Conclusions

This paper extended the original GMC to an interval PGMC, which is capable of controlling a class of non-linear systems with interval parameters with guaranteed output stability. The incorporation of the multistep prediction reduced the control signal variations caused by the ‘near-sighted’ nature associated with the original GMC. The adaptive factor was used in conjunction with parameter intervals to improve the closed-loop system performance. Theoretical foundations of the proposed interval PGMC were developed using interval arithmetic, which established the condition to guarantee the system’s output stability defined by a pre-specified output range. Simulation and experimental results of an arc welding process control verified the effectiveness of the proposed algorithm.

Acknowledgement

This research is funded by the National Science Foundation under Grant DMI-0114982.

References


